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Theory of (Vector Valued) Fourier Hyperfunctions. Their Realization as Boundary Values of (Vector Valued) Slowly Increasing Holomorphic Functions (IV)

Dedicated to Professor Nobuhiko Tatsuuma on his 60th birthday

By

Yoshifumi Ito

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Introduction

This paper is the fourth part of this series of papers, which includes Chapter 10. For the outline of this paper, see "Contents" in the first part of this series of papers [37]. Here we note that "isomorphisms" usually mean topological ones without explicit mention for the contrary. For References we refer to the lists of references at the end of papers [37], [38], [40] and this one.

Chapter 10. Case of the sheaf ${}^E\mathcal{O}^{\natural}$

10.1. The Dolbeault-Grothendieck resolution of ${}^E\mathcal{O}^{\natural}$

In this section we will construct a soft resolution of ${}^E\mathcal{O}^{\natural}$. In this chapter we always assume that E is a Fréchet space whose topology is defined by a family $\mathscr{T} = \mathscr{T}_E$ of continuous seminorms of E.

At first we will define sheaves ${}^E\mathcal{O}^{\natural}$ and ${}^E\mathscr{E}^{\natural}$.

Definition 10.1.1 (The sheaf ${}^E\mathcal{O}^{\natural}$ of germs of partially slowly increasing *E*-valued holomorphic functions). *We define the sheaf ${}^E\mathcal{O}^{\natural}$ to be the sheafification of the presheaf $\{\mathcal{O}^{\natural}(\Omega; E)\}$, where, for an open set Ω in $\boldsymbol{H}^n$, the module $\mathcal{O}^{\natural}(\Omega; E)$ is defined as follows*:

$$\mathcal{O}^{\natural}(\Omega; E) = \{f \in \mathcal{O}(\Omega \cap \boldsymbol{C}^{|n|}; E);\ \text{for any positive } \varepsilon \text{ and any compact set } K \text{ in } \Omega \text{ and any } q \in \mathscr{T},\ \sup\{q(f(z))e(-\varepsilon|z|);\ z \in K \cap \boldsymbol{C}^{|n|}\} < \infty \text{ holds}\}.$$

We call this sheaf ${}^E\mathcal{O}^{\natural}$ the sheaf of germs of partially slowly increasing E-valued holomorphic functions.

**Definition 10.1.2 (The sheaf ${}^E\mathscr{E}^{\natural}$ of germs of partially increasing *E-valued* C^{∞}-

functions). *We define* ${}^{E}\mathscr{E}^{\natural}$ *to be the sheafification of the presheaf* $\{\mathscr{E}^{\natural}(\Omega; E)\}$, *where, for an open set* Ω *in* $\boldsymbol{H}^n$, *the module* $\mathscr{E}^{\natural}(\Omega; E)$ *is defined as follows*:

> $\mathscr{E}^{\natural}(\Omega; E) = \{f \in \mathscr{E}(\Omega \cap \boldsymbol{C}^{|n|}; E)$; *for any positive* ε *and any compact set* K *in* Ω *and any* $\alpha \in \bar{\boldsymbol{N}}^{2|n|}$ *and any* $q \in \mathscr{T}$, $\sup\{q(f^{(\alpha)}(z))e(-\varepsilon|z|); z \in K \cap \boldsymbol{C}^{|n|}\} < \infty$ *holds*$\}$.

Then the sheaf ${}^{E}\mathscr{E}^{\natural}$ is a soft Fréchet sheaf and we have the following.

Theorem 10.1.3 (The Dolbeault-Grothendieck resolution of ${}^{E}\mathcal{O}^{\natural,p}$**).** *The sequence of sheaves*

$$0 \longrightarrow {}^{E}\mathcal{O}^{\natural,p}|X \longrightarrow {}^{E}\mathscr{E}^{\natural,p,0}|X \xrightarrow{\bar{\partial}} {}^{E}\mathscr{E}^{\natural,p,1}|X \xrightarrow{\bar{\partial}} \cdots \xrightarrow{\bar{\partial}} {}^{E}\mathscr{E}^{\natural,p,|n|}|X \longrightarrow 0$$

is exact, where $X = \operatorname{int}\{z \in \boldsymbol{C}^{|n|}; ||\operatorname{Im} z''| - |\operatorname{Re} z''| < d, |\operatorname{Im} z''| < 1 + |\operatorname{Re} z''|/\sqrt{3}\}^a$ *for some* $d > 0$.

Proof. The exactness of the sequence

$$0 \longrightarrow {}^{E}\mathcal{O}^{\natural,p}|X \longrightarrow {}^{E}\mathscr{E}^{\natural,p,0}|X \xrightarrow{\bar{\partial}} {}^{E}\mathscr{E}^{\natural,p,1}|X$$

is evident.

Next the exactness of the sequence

$${}^{E}\mathscr{E}^{\natural,p,0}|X \xrightarrow{\bar{\partial}} {}^{E}\mathscr{E}^{\natural,p,1}|X \xrightarrow{\bar{\partial}} \cdots \xrightarrow{\bar{\partial}} {}^{E}\mathscr{E}^{\natural,p,|n|}|X \longrightarrow 0$$

follows from the following Lemma 10.1.4. This completes the proof of Theorem 10.1.3. (Q.E.D.)

Lemma 10.1.4. *We use the notations in Theorem* 10.1.3. *Let* Ω *be an* $\mathcal{O}^{\natural}$-*pseudoconvex open set in* X. *Then the equation* $\bar{\partial}u = f$ *has a solution* $u \in \mathscr{E}^{\natural,p,q}(\Omega; E)$ *for every* $f \in \mathscr{E}^{\natural,p,q+1}(\Omega; E)$ *such that* $\bar{\partial}f = 0$. *Here* $p, q \geqq 0$.

Proof. If we put

$$Z^{\natural,p,q+1}(\Omega) = \{f \in \mathscr{E}^{\natural,p,q+1}(\Omega); \bar{\partial}f = 0\}$$

and

$$Z^{\natural,p,q+1}(\Omega; E) = \{f \in \mathscr{E}^{\natural,p,q+1}(\Omega; E); \bar{\partial}f = 0\},$$

then $Z^{\natural,p,q+1}(\Omega)$ is a nuclear Fréchet space and

$$Z^{\natural,p,q+1}(\Omega; E) \cong Z^{\natural,p,q+1}(\Omega) \hat{\otimes} E$$

holds. By virtue of Corollary 2 of Theorem 9.2.8, we have an exact sequence

$$\mathscr{E}^{\natural,p,q}(\Omega) \xrightarrow{\bar{\partial}} Z^{\natural,p,q+1}(\Omega) \longrightarrow 0$$

for the $\mathcal{O}^{\natural}$-pseudoconvex open set Ω in X. Then, since we have also

$$\mathscr{E}^{\natural,p,q}(\Omega; E) \cong \mathscr{E}^{\natural,p,q}(\Omega) \hat{\otimes} E,$$

we have an exact sequence

$$\mathscr{E}^{\natural,p,q}(\Omega; E) \xrightarrow{\bar{\partial}} Z^{\natural,p,q+1}(\Omega; E) \longrightarrow 0$$

by virtue of Trèves [36], Proposition 43.9. (Q.E.D.)

Corollary. *We use the notations in Theorem* 10.1.3. *For an open set Ω in X, we have the following isomorphism*:

$$H^q(\Omega, {}^E\mathcal{O}^{\natural,p}) \cong \{f \in \mathscr{E}^{\natural,p,q}(\Omega; E);\ \bar{\partial} f = 0\}/\{\bar{\partial} g;\ g \in \mathscr{E}^{\natural,p,q-1}(\Omega; E)\},$$

$(p \geqq 0$ *and* $q \geqq 1)$.

Proof. It follows from Theorem 10.1.3 and Komatsu [21], Theorems II.2.9 and II.2.19. (Q.E.D.)

10.2. The Oka-Cartan-Kawai Theorem B

We will prove the Oka-Cartan-Kawai Theorem B for the sheaf ${}^E\mathcal{O}^{\natural}$.

Theorem 10.2.1 (The Oka-Cartan-Kawai Theorem B). *Let X be as in Theorem* 10.1.3. *For every $\mathcal{O}^{\natural}$-pseudoconvex open set Ω in X, we have* $H^q(\Omega, {}^E\mathcal{O}^{\natural,p}) = 0$ *for* $p \geqq 0$ *and* $q \geqq 1$.

Proof. Since we have, by the Oka-Cartan-Kawai Theorem B for $\mathcal{O}^{\natural}$,

$$H^q(\Omega, \mathcal{O}^{\natural,p}) = 0, \qquad p \geqq 0 \text{ and } q \geqq 1,$$

the complex obtained from Theorem 9.2.8:

$$\mathscr{E}^{\natural,p,0}(\Omega) \xrightarrow{\bar{\partial}} \mathscr{E}^{\natural,p,1}(\Omega) \xrightarrow{\bar{\partial}} \cdots \xrightarrow{\bar{\partial}} \mathscr{E}^{\natural,p,|n|}(\Omega) \longrightarrow 0$$

is exact. Since $\mathscr{E}^{\natural,p,q}(\Omega)$'s are all nuclear Fréchet spaces and E is a Fréchet space, the complex

$$\mathscr{E}^{\natural,p,0}(\Omega; E) \xrightarrow{\bar{\partial}} \mathscr{E}^{\natural,p,1}(\Omega; E) \xrightarrow{\bar{\partial}} \cdots \xrightarrow{\bar{\partial}} \mathscr{E}^{\natural,p,|n|}(\Omega; E) \longrightarrow 0$$

is also exact by virtue of the isomorphism

$$\mathscr{E}^{\natural,p,q}(\Omega; E) \cong \mathscr{E}^{\natural,p,q}(\Omega) \hat{\otimes} E$$

and Ion-Kawai [5], Theorem 1.10. Hence we obtain

$$H^q(\Omega, {}^E\mathcal{O}^{\natural,p}) = 0, \qquad p \geqq 0 \text{ and } q \geqq 1.$$

This completes the proof. (Q.E.D.)

Corollary. *We use the notations in the theorem* 10.2.1. *Let Ω be an $\mathcal{O}^\natural$-pseudoconvex open set in X. Then the equation $\bar{\partial}u = f$ has a solution $u \in \mathcal{E}^{\natural,p,q}(\Omega; E)$ for every $f \in \mathcal{E}^{\natural,p,q+1}(\Omega; E)$ such that $\bar{\partial}f = 0$. Here p and q are nonnegative integers.*

Proof. It follows from Theorem 10.2.1 and Corollary to Theorem 10.1.3. (Q.E.D.)

10.3. The Malgrange Theorem

We will prove the Malgrange Theorem for the sheaf ${}^E\mathcal{O}^\natural$.

Theorem 10.3.1. *Let X be as in Theorem* 10.1.3. *Let Ω be an open set in X. Then we have $H^{|n|}(\Omega, {}^E\mathcal{O}^\natural) = 0$.*

Proof. By virtue of Theorems 9.2.8 and 9.3.1, we have an exact sequence

$$\mathcal{E}^{\natural,0,|n|-1}(\Omega) \xrightarrow{\bar{\partial}} \mathcal{E}^{\natural,0,|n|}(\Omega) \longrightarrow 0.$$

Thus, by Trèves [36], Proposition 43.9, we have the exact sequence

$$\mathcal{E}^{\natural,0,|n|-1}(\Omega) \hat{\otimes} E \xrightarrow{\bar{\partial}} \mathcal{E}^{\natural,0,|n|}(\Omega) \hat{\otimes} E \longrightarrow 0$$

or

$$\mathcal{E}^{\natural,0,|n|-1}(\Omega; E) \xrightarrow{\bar{\partial}} \mathcal{E}^{\natural,0,|n|}(\Omega; E) \longrightarrow 0.$$

Hence we obtain the conclusion. (Q.E.D.)

Corollary. Flabby $\dim {}^E\mathcal{O}^\natural \leqq |n|$.

10.4. The Serre Duality Theorem

In this section we will prove the Serre Duality Theorem for the sheaves ${}^E\mathcal{O}^\natural$ and $\mathcal{O}_\natural$.

Theorem 10.4.1. *Let X be as in Theorem* 9.4.1 *and Ω an open set in X such that* $\dim H^p(\Omega, \mathcal{O}^\natural) < \infty$ *holds for each $p \geqq 1$. Then we have the isomorphism*

$H^p(\Omega, {}^E\mathcal{O}^\natural) \cong L(H_c^{|n|-p}(\Omega, \mathcal{O}_\natural); E)$ *for* $0 \leqq p \leqq |n|$.

Proof. By a similar method to Junker [15], Lemma 3.5, we can obtain the isomorphism $H^p(\Omega, {}^E\mathcal{O}^\natural) \cong H^p(\Omega, \mathcal{O}^\natural) \hat{\otimes} E$. Then, by Theorem 9.4.1, we have the following isomorphisms

$$H^p(\Omega, {}^E\mathcal{O}^\natural) \cong H^p(\Omega, \mathcal{O}^\natural) \hat{\otimes} E \cong [H_c^{|n|-p}(\Omega, \mathcal{O}_\natural)]' \hat{\otimes} E \cong L(H_c^{|n|-p}(\Omega, \mathcal{O}_\natural); E).$$

(Q.E.D.)

Here we note that the Remark at the end of Section 9.4 also works in this case.

10.5. The Martineau-Harvey Theorem

In this section we will prove the Martineau-Harvey Theorem for the sheaves ${}^E\mathcal{O}^\natural$ and $\mathcal{O}_\natural$.

Theorem 10.5.1. *Let X, K and Ω be as in Theorem* 9.5.1. *Then*

(1) $H_K^p(\Omega, {}^E\mathcal{O}^\natural) = 0, \quad (p \neq |n|)$.

(2) *If* $|n| \geqq 2$, *we have algebraic isomorphisms*

$$H_K^{|n|}(\Omega, {}^E\mathcal{O}^\natural) \cong H^{|n|-1}(\Omega \backslash K, {}^E\mathcal{O}^\natural) \cong L(\mathcal{O}_\natural(K); E).$$

(3) *If* $|n| = 1$, *we have the algebraic isomorphism*

$$H_K^1(\Omega, {}^E\mathcal{O}^\natural) \cong \mathcal{O}^\natural(\Omega \backslash K; E)/\mathcal{O}^\natural(\Omega; E).$$

Proof. At first assume $|n| \geqq 2$. By virtue of the excision theorem, $H_K^p(\Omega, {}^E\mathcal{O}^\natural)$ is independent of an open neighborhood Ω of K. So, we may assume that Ω is the $\mathcal{O}^\natural$-pseudoconvex open neighborhood in the assumptions in this theorem. Then, in the long exact sequence of cohomology groups (cf. Komatsu [21], Theorem II.3.2):

$$\begin{aligned} 0 &\longrightarrow H_K^0(\Omega, {}^E\mathcal{O}^\natural) \longrightarrow H^0(\Omega, {}^E\mathcal{O}^\natural) \longrightarrow H^0(\Omega \backslash K, {}^E\mathcal{O}^\natural) \\ &\longrightarrow H_K^1(\Omega, {}^E\mathcal{O}^\natural) \longrightarrow \cdots \\ &\longrightarrow H_K^{|n|}(\Omega, {}^E\mathcal{O}^\natural) \longrightarrow H^{|n|}(\Omega, {}^E\mathcal{O}^\natural) \longrightarrow H^{|n|}(\Omega \backslash K, {}^E\mathcal{O}^\natural) \longrightarrow \cdots, \end{aligned}$$

we have $H^p(\Omega, {}^E\mathcal{O}^\natural) = 0$ for each $p \geqq 1$, and $H_K^0(\Omega, {}^E\mathcal{O}^\natural) = 0$ by the unique continuation theorem. Hence we have algebraic isomorphisms

$$\begin{aligned} &H_K^1(\Omega, {}^E\mathcal{O}^\natural) \cong \mathcal{O}^\natural(\Omega \backslash K; E)/\mathcal{O}^\natural(\Omega; E), \\ &H_K^p(\Omega, {}^E\mathcal{O}^\natural) \cong H^{p-1}(\Omega \backslash K, {}^E\mathcal{O}^\natural), \qquad p \geqq 2. \end{aligned}$$

But, by a similar method to Junker [15], Lemma 3.5, we have isomorphisms

$H^p(V, {}^E\mathcal{O}^{\natural}) \cong H^p(V, \mathcal{O}^{\natural}) \hat{\otimes} E$, $(0 \leqq p \leqq |n|)$, where V is an open set in X. Since we have isomorphisms

$$\mathcal{O}^{\natural}(\Omega \backslash K; E) \cong \mathcal{O}^{\natural}(\Omega \backslash K) \hat{\otimes} E \cong \mathcal{O}^{\natural}(\Omega) \hat{\otimes} E \cong \mathcal{O}^{\natural}(\Omega; E),$$

we have $H^1_K(\Omega, {}^E\mathcal{O}^{\natural}) = 0$. For each $p \geqq 2$, $p \neq |n|$, we have, by the Martineau-Harvey Theorem for $\mathcal{O}^{\natural}$,

$$H^p_K(\Omega, {}^E\mathcal{O}^{\natural}) \cong H^{p-1}(\Omega \backslash K, {}^E\mathcal{O}^{\natural}) \cong H^{p-1}(\Omega \backslash K, \mathcal{O}^{\natural}) \hat{\otimes} E = 0.$$

Now, by Proposition 9.5.2, we have the topological isomorphism

$$\mathcal{O}_{\natural}(K) \cong H^1_c(\Omega \backslash K, \mathcal{O}_{\natural}).$$

Thus, by Theorem 10.4.1, we have algebraic isomorphisms

$$\begin{aligned} H^{|n|}_K(\Omega, {}^E\mathcal{O}^{\natural}) &\cong H^{|n|-1}(\Omega \backslash K, {}^E\mathcal{O}^{\natural}) \\ &\cong L(H^1_c(\Omega \backslash K, \mathcal{O}_{\natural}); E) \cong L(\mathcal{O}_{\natural}(K); E). \end{aligned}$$

At last we will prove the case $|n| = 1$. In this case, we have the conclusion by virtue of the long excat sequence of relative cohomology groups, the Oka-Cartan-Kawai Theorem B and the Malgrange Theorem. (Q.E.D.)

10.6. The Sato Theorem

In this section we will prove the pure-codimensionality of $\tilde{\boldsymbol{D}}^n$ with respect to ${}^E\mathcal{O}^{\natural}$. Then we will realize E-valued partial modified Fourier hyperfunctions as "boundary values" of E-valued partially slowly increasing holomorphic functions or as (relative) cohomology classes of E-valued partially slowly increasing holomorphic functions. Thereby two realizations of E-valued partial modified Fourier hyperfunctions are identified (cf. Theorem 10.6.1, (3) below).

Theorem 10.6.1 (The Sato Theorem). *Assume* $|n| \geqq 2$. *Then we have the following*:

(1) $\tilde{\boldsymbol{D}}^n$ *is purely* $|n|$*-codimensional with respect to* ${}^E\mathcal{O}^{\natural}$.

(2) *The presheaf over* $\tilde{\boldsymbol{D}}^n$, $\Omega \rightarrow H^{|n|}_{\Omega}(V, {}^E\mathcal{O}^{\natural})$ *is a flabby sheaf, where* Ω *is an open set in* $\tilde{\boldsymbol{D}}^n$ *and* V *an open set in* X *which contains* Ω *as its closed subset. Here* X *is as in Theorem* 10.5.1.

(3) *This sheaf* (2) *is isomorphic to the sheaf* ${}^E(\mathscr{B}\mathscr{Q})$ *of E-valued partial modified Fourier hyperfunctions.*

Proof. (1) We have to prove the vanishing of the derived sheaf $\mathscr{H}^p_{\tilde{\boldsymbol{D}}^n}({}^E\mathcal{O}^{\natural})$ for $p \neq |n|$. This is local in nature. Thus, it is sufficient to prove $H^p_{\Omega}(V, {}^E\mathcal{O}^{\natural}) = 0$, $(p \neq |n|)$, for a relatively compact open set Ω in $\tilde{\boldsymbol{D}}^n$. Thus, by the excision theorem, we may assume that V is an $\mathcal{O}^{\natural}$-pseudoconvex open set in X. Consider the following exact sequence of relative cohomology groups

$$\begin{aligned}0 &\longrightarrow H^0_{\partial\Omega}(V, {}^E\mathcal{O}^\natural) \longrightarrow H^0_{\Omega^a}(V, {}^E\mathcal{O}^\natural) \longrightarrow H^0_\Omega(V, {}^E\mathcal{O}^\natural)\\ &\longrightarrow H^1_{\partial\Omega}(V, {}^E\mathcal{O}^\natural) \longrightarrow \cdots \longrightarrow H^{|n|-1}_\Omega(V, {}^E\mathcal{O}^\natural)\\ &\longrightarrow H^{|n|}_{\partial\Omega}(V, {}^E\mathcal{O}^\natural) \longrightarrow H^{|n|}_{\Omega^a}(V, {}^E\mathcal{O}^\natural) \longrightarrow H^{|n|}_\Omega(V, {}^E\mathcal{O}^\natural)\\ &\longrightarrow H^{|n|+1}_{\partial\Omega}(V, {}^E\mathcal{O}^\natural) \longrightarrow \cdots.\end{aligned}$$

By Theorems 9.1.8 and 10.5.1, we may conclude that $H^p_{\partial\Omega}(V, {}^E\mathcal{O}^\natural) = H^p_{\Omega^a}(V, {}^E\mathcal{O}^\natural) = 0$ for every $p \neq |n|$. So that, we have $H^p_\Omega(V, {}^E\mathcal{O}^\natural) = 0$ for every $p \neq |n| - 1$, $|n|$. On the other hand, by Theorems 9.1.8 and 10.5.1, we also have the exact sequence

$$0 \longrightarrow H^{|n|-1}_\Omega(V, {}^E\mathcal{O}^\natural) \longrightarrow L(\mathscr{A}_\natural(\partial\Omega); E) \xrightarrow{j} L(\mathscr{A}_\natural(\Omega^a); E).$$

Since j is injective, we have $H^{|n|-1}_\Omega(V, {}^E\mathcal{O}^\natural) = 0$.

(2) By (1) and Komatsu [21], Theorem II.3.18, we have the conclusion.

(3) We have only to prove this isomorphism stalkswise. This is local in nature. By the proof of (1), we have the exact sequence for a relatively compact open set Ω in $\tilde{\boldsymbol{D}}^n$

$$0 \longrightarrow H^{|n|}_{\partial\Omega}(V, {}^E\mathcal{O}^\natural) \longrightarrow H^{|n|}_{\Omega^a}(V, {}^E\mathcal{O}^\natural) \longrightarrow H^{|n|}_\Omega(V, {}^E\mathcal{O}^\natural) \longrightarrow 0.$$

Since, by the Martineau-Harvey Theorem, we have algebraic isomorphisms

$$\begin{aligned}H^{|n|}_{\partial\Omega}(V, {}^E\mathcal{O}^\natural) &\cong L(\mathscr{A}_\natural(\partial\Omega); E),\\ H^{|n|}_{\Omega^a}(V, {}^E\mathcal{O}^\natural) &\cong L(\mathscr{A}_\natural(\Omega^a); E),\end{aligned}$$

we obtain the algebraic isomorphism

$$H^{|n|}_\Omega(V, {}^E\mathcal{O}^\natural) \cong L(\mathscr{A}_\natural(\Omega^a); E)/L(\mathscr{A}^\natural(\partial\Omega); E) = \mathscr{B}\mathscr{Q}(\Omega; E).$$

Thus the sheaf $\Omega \to H^{|n|}_\Omega(V, {}^E\mathcal{O}^\natural)$ is isomorphic to the sheaf ${}^E(\mathscr{B}\mathscr{Q})$ of E-valued partial modified Fourier hyperfunctions over $\tilde{\boldsymbol{D}}^n$. (Q.E.D.)

Corollary. *Assume* $|n| \geqq 2$. *We use the notations in Theorem* 10.6.1. *Then we have the algebraic isomorphism* $H^{|n|}_\Omega(V, {}^E\mathcal{O}^\natural) \cong H^{|n|-1}(V\backslash\Omega, {}^E\mathcal{O}^\natural)$.

Next consider the case $|n| = 1$. This case is either $n = (1, 0)$ or $n = (0, 1)$. The case $n = (1, 0)$ results in the case of E-valued Sato hyperfunctions and the case $n = (0, 1)$ results in the case of E-valued modified Fourier hyperfunctions. In these cases the Sato Theorem is also true except that, in the case $n = (0, 1)$, the assertion (3) is not yet proved.

In similar notations to Theorem 9.6.2, we have the following.

Theorem 10.6.2. $H^{|n|}_{\Omega}(V, {}^E\mathcal{O}^\natural) \cong H^{|n|}(\mathfrak{V}, \mathfrak{V}, {}^E\mathcal{O}^\natural) \cong \mathcal{O}^\natural(\bigcap_j V_j;\ E)/\sum_{j=1}^{|n|}\mathcal{O}^\natural(\bigcap_{i\neq j} V_i;\ E)$ *hold algebraically.*

At last we will realize partial modified Fourier analytic linear mappings with certain compact carrier as (relative) cohomology classes with coefficients in ${}^E\mathcal{O}^\natural$.

Assume $|n| \geqq 2$. Let X be as in Theorem 10.5.1 and K a compact set in X of the form $K = K_1 \times \cdots \times K_{|n|}$ with compact sets K_j in $\boldsymbol{C}$ for $j = 1, 2, \dots, n_1$ and in $\boldsymbol{E}$ for $j = n_1 + 1, \dots, |n|$. Assume that K admits a fundamental system of $\mathcal{O}^\natural$-pseudoconvex open neighborhoods. Then we have

$$H^p(K, \mathcal{O}_\natural) = 0 \qquad \text{for every } p > 0.$$

By virtue of the Martineau-Harvey Theorem, there exists the algebraic isomorphism

$$\mathcal{O}_\natural(K;\ E) \cong H^{|n|}_K(\Omega, {}^E\mathcal{O}^\natural).$$

Here Ω denotes an open neighborhood of K. Further assume that there exists an $\mathcal{O}^\natural$-pseudoconvex open neighborhood Ω of K such that

$$\Omega_j = \Omega \backslash \{z \in \boldsymbol{C}^{|n|};\ z_j \in K_j \cap \boldsymbol{C}\}^a$$

is also an $\mathcal{O}^\natural$-pseudoconvex open set for $j = 1, 2, \dots, |n|$. If $K \subset \tilde{\boldsymbol{D}}^n$ holds, all the assumptions imposed on K in the above are satisfied. Put $\Omega_0 = \Omega$. Then $\mathfrak{U} = \{\Omega_0, \Omega_1, \dots, \Omega_{|n|}\}$ and $\mathfrak{U}' = \{\Omega_1, \Omega_2, \dots, \Omega_{|n|}\}$ form acyclic coverings of Ω and $\Omega \backslash K$. Set

$$\Omega \# K = \bigcap_{j=1}^{|n|} \Omega_j, \qquad \Omega^j = \bigcap_{i\neq j} \Omega_i.$$

Let $\sum_j \mathcal{O}^\natural(\Omega^j;\ E)$ be the image in $\mathcal{O}^\natural(\Omega \# K;\ E)$ of $\prod_{j=1}^{|n|} \mathcal{O}^\natural(\Omega^j;\ E)$ by the mapping

$$(f_j)_{j=1}^{|n|} \longrightarrow \sum_{j=1}^{|n|} (-1)^{j+1} f_j',$$

where f_j' denotes the restriction of f_j to $\Omega \# K$.

Then, by a similar way to Theorem 10.6.2, we have the following.

Theorem 10.6.3. *We use the notations as above. Then we have the algebraic isomorphisms*

$$\mathcal{O}'_\natural(K;\ E) \cong H^{|n|}_K(\Omega, {}^E\mathcal{O}^\natural) \cong H^{|n|}(\mathfrak{U}, \mathfrak{U}', {}^E\mathcal{O}^\natural) \cong \mathcal{O}^\natural(\Omega \# K;\ E)/\textstyle\sum_j \mathcal{O}^\natural(\Omega^j;\ E).$$

Department of Mathematics
College of General Education
Tokushima University

References

[1]–[39], See References in Y. Ito [37], [38] and [40].

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