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Errata of "Theory of (Vector Valued) Fourier Hyperfunctions. Their Realization as Boundary Values of (Vector Valued) Slowly Increasing Holomorphic Functions, (III), *J. Math. Tokushima Univ.*, 23 (1989), 23–38".

By

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p. 23 ↓9 "Contents" → "Contents"

↓10 "isomorphisms" → "isomorphisms"

p. 27 ↓18 $\mathscr{L}^{\natural.p,q}(\Omega) \to \mathscr{L}^{\natural,p,q}(\Omega)$

p. 32 The proof of Theorem 9.3.1 should be changed into the following:

Proof of Theorem 9.3.1. By virtue of Corollary 1 to Theorem 9.2.3, we have only to prove the exactness of the sequence

$$\mathscr{L}^{\natural,0,|n|-1}(\Omega) \xrightarrow{\bar{\partial}} \mathscr{L}^{\natural,0,|n|}(\Omega) \longrightarrow 0$$

in the notations of Theorem 9.2.3. This is equivalent to proving the exactness of the sequence

$$L^{\natural,0,|n|-1}(\Omega) \xrightarrow{\bar{\partial}} L^{\natural,0,|n|}(\Omega) \longrightarrow 0.$$

By virtue of the Serre-Komatsu duality theorem for FS*-spaces, it suffices to show the injectiveness and closedrangeness of $-\bar{\partial} = (\bar{\partial})'$ in the dual sequence

$$\text{(I)} \qquad L^{0,1}_{\natural,c}(\Omega) \xleftarrow{-\bar{\partial}} L^{0,0}_{\natural,c}(\Omega) \longleftarrow 0.$$

Here $L^{p,q}_{\natural,c}(\Omega)$ denotes the space of sections with compact support of $L^{p,q}_{\natural}$ on Ω. Since $-\bar{\partial}$ is elliptic, its injectivity is an immediate consequence of the unique continuation property. Now we will prove its closedrangeness. This is surely true if Ω is replaced by a larger $\mathcal{O}^{\natural}$-pseudoconvex open set $\Omega(\subset X)$ containing Ω because then $H^p(\tilde{\Omega}, \mathcal{O}^{\natural}) = 0$ for $p \geqq 1$. Thus the problem reduces to the estimation of support of the solution of the system $-\bar{\partial}u = f$ employing the principle of analytic continuation. Note that in a DFS*-space we can test the closedness by converging sequences. So assume that $u_k \in L^{0,0}_{\natural,c}(\Omega)$ and $-\bar{\partial}u_k \to f$ in

$L^{0,1}_{\natural,c}(\Omega)$. Then the convergence takes place in $L^{0,1}_{\natural,c}(\tilde{\Omega})$. Hence, by the closed range property of

$$\text{(II)} \qquad L^{0,1}_{\natural,c}(\tilde{\Omega}) \xleftarrow{-\bar{\partial}} L^{0,0}_{\natural,c}(\tilde{\Omega}) \longleftarrow 0,$$

we can find $u \in L^{0,0}_{\natural,c}(\tilde{\Omega})$ such that $-\bar{\partial}u = f$. Since u is holomorphic outside $\mathrm{supp}(f)$, we see that $\mathrm{supp}(u) \subset K$ by virtue of the unique continuation property where K is a compact subset of $\tilde{\Omega}$ which may contain some connected components of $X \setminus \Omega$ contained in $\tilde{\Omega}$ but on which u is holomorphic. It remains to show that u is in fact zero on these components. Again by the closedness of $-\bar{\partial}$ in (II) and the stability of DFS*-property for closed subspaces, we can apply the open mapping theorem and find some sequence v_k, $v \in L^{0,0}_{\natural,c}(\tilde{\Omega})$ such that $-\bar{\partial}v_k = -\bar{\partial}u_k$, $-\bar{\partial}v = -\bar{\partial}u$ and that $v_k \to v$ in $L^{0,0}_{\natural,c}(\tilde{\Omega})$. But by the condition of support, we must have $v_k = u_k$, $v = u$. Hence $u_k \to u$ in $L^{0,0}_{\natural,c}(\tilde{\Omega})$. But by the local character of the topology of this space, we thus conclude that $\mathrm{supp}(u)$ must be contained in Ω. This shows that (I) has the closed range. Q.E.D.

p. 35 $\downarrow 1 \sim \downarrow 3$ However, $\cdots\cdots$ is closed. $\longrightarrow$

Here we have the algebraic isomorphism

$$H^{|n|}_c(\Omega \setminus K, \mathcal{O}_\natural) \cong H^{|n|}_c(\Omega, \mathcal{O}_\natural),$$

which is in fact a topological isomorphism as seen in the Proposition 9.5.2 below. By definition, we have topological isomorphisms

$$H^{|n|}_c(\Omega \setminus K, O_\natural) \cong \mathscr{L}^{0,|n|}_{\natural,c}(\Omega \setminus K)/\mathrm{Im}(-\bar{\partial}^{\Omega\setminus K}_{|n|-1}),$$
$$H^{|n|}_c(\Omega, \mathcal{O}_\natural) \cong \mathscr{L}^{0,|n|}_{\natural,c}(\Omega)/\mathrm{Im}(-\bar{\partial}^{\Omega}_{|n|-1}).$$

Hence we have the following exact sequence

$$0 \longrightarrow \mathrm{Im}(-\bar{\partial}^{\Omega\setminus K}_{|n|-1}) \longrightarrow \mathscr{L}^{0,|n|}_{\natural,c}(\Omega \setminus K) \xrightarrow{\alpha} H^{|n|}_c(\Omega, \mathcal{O}_\natural).$$

Here, since $H^1(\Omega, \mathcal{O}^\natural) = 0$ holds, $\bar{\partial}^{\Omega}_0$ is of closed range in the dual complexes as above for Ω. Hence, by the closed range theorem, $-\bar{\partial}^{\Omega}_{|n|-1}$ is of closed range. Hence $H^{|n|}_c(\Omega, \mathcal{O}_\natural)$ is Hausdorff. Thus the set $\{0\}$ in $H^{|n|}_c(\Omega, \mathcal{O}_\natural)$ is closed. Since α is continuous with respect to the natural topologies, $\mathrm{Ker}(\alpha) = \mathrm{Im}(-\bar{\partial}^{\Omega\setminus K}_{|n|-1})$ is closed.

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