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Sato Hyperfunctions Valued in a Locally Convex Space

By

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Abstract

In this article, we realize, by the duality method, Sato hyperfunctions valued in a locally convex topological vector space, which is not necessarily a Fréchet space. We prove analogs of Schwartz's Kernel Theorem for analytic-linear mappings and vector-valued Sato hyperfunctions. Further we define several operations on analytic-linear mappings and vector-valued Sato hyperfunctions.

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Introduction

Since, in 1959−1960, Sato established the theory of Sato hyperfunctions, some authors have tried to extend this theory to the vector-valued case(cf. Ion-Kawai[5] and Ito[7], [9] and [10]). Until now the case of Fréchet-space-valued Sato hyperfunctions was considered as the limiting case because of the difficulty of the theory of functions of several complex variables.

In this article we show that we can realize Sato hyperfunctions valued in a locally convex topological vector space E by the duality method. Here E is not necessarily a Fréchet space. This realization is difficult by the algebro-analytic method if E is not a Fréchet space(cf. Ion-Kawai[5] and Ito[7], [9]). In fact, there exist counterexamples of Itano and Vogt[6], [16], which show that there exist some vector-valued distributions that cannot be realized as boundary values of holomorphic functions. Let $\mathcal{A}(K)$ be the space of all real-analytic functions on some neighborhoods of K when K is a compact set in a real-analytic manifold M countable at infinity. Then $\mathcal{A}(K)$ is a nuclear DFS-space. Then we consider the space $\mathcal{A}'(K; E) = L(\mathcal{A}(K), E)$ of all continuous linear mappings from $\mathcal{A}(K)$ into E, endowed with the topology of uniform convergence on each bounded set in $\mathcal{A}(K)$. Then, for a relatively compact open set Ω in M, we define the space $\mathcal{B}(\Omega; E) = \mathcal{A}'(\Omega^{\mathrm{cl}}; E)/\mathcal{A}'(\partial\Omega; E)$ and call each element in $\mathcal{B}(\Omega; E)$ an E-valued Sato hyperfunction on Ω. If we put $\mathcal{B}_1(\Omega; E) = 0$ for an open set Ω in M which is not relatively compact and put $\mathcal{B}_1(\Omega; E) = \mathcal{B}(\Omega; E)$ for a relatively compact open set Ω in M and consider natural restriction mappings, then the family $\{\mathcal{B}_1(\Omega; E); \Omega \text{ is an open set in } M\}$ becomes a presheaf, but in general, it is not a sheaf. We denote by ${}^E\mathcal{B}$ the sheafification of this presheaf and call it the sheaf of E-valued Sato hyperfunctions over M. If E is a Fréchet space, the presheaf $\{\mathcal{B}(\omega; E); \omega \text{ is an open subset of } \Omega\}$ becomes a sheaf over a relatively compact open set Ω in M. By virtue of this fact, we can see that the sheaf ${}^E\mathcal{B}$ is flabby (cf. Ito[7], [9] and [10]). But if E is not a Fréchet space, we can not see that the sheaf ${}^E\mathcal{B}$ is flabby. This is an open problem.

We can prove analogs of Schwartz's Kernel Theorem for analytic-linear mappings and E-valued Sato hyperfunctions if E is complete. Further we define several operations on analytic-linear mappings and E-valued Sato hyperfunctions.

In section 1, we recall some properties of holomorphic functions and real-analytic functions.

In section 2, we define analytic-linear mappings and mention some of their properties.

In section 3, we define E-valued Sato hyperfunctions and mention some of their properties.

In section 4, we prove analogs of Schwartz's Kernel Theorem for analytic-linear mappings valued in a complete, locally convex, topological vector space and define several operations on analytic-linear mappings.

In section 5, we prove analogs of Schwartz's Kernel Theorem for E-valued Sato hyperfunctions when E is complete, and define several operations on vector-valued Sato hyperfunctions.

1. Holomorphic functions and real-analytic functions

Let M be an n-dimensional and real-analytic manifold countable at infinity and X a complexification of M. We denote by $\mathcal{O}$ the sheaf of germs of holomorphic functions over X and by $\mathcal{A}$ the sheaf of germs of real-analytic functions over M. Then $\mathcal{A} = \mathcal{O}|_M$ holds.

Then we have the following.

Theorem 1.1(the Oka-Cartan Theorem B). *Let Ω be a Stein open set in*

X. *Then we have* $H^p(\Omega, \mathcal{O}) = 0,\ (p > 0)$.

Proof. See Hörmander[4], Corollary 7.4.2, p.182. Q.E.D.

Theorem 1.2(Malgrange). *For every compact set* K *in* M*, we have*

$$H^p(K, \mathcal{A}) = 0,\ (p > 0).$$

Proof. We know, by virtue of Grauert's Theorem (cf. Grauert[3]), that K has a fundamental system of Stein open neighborhoods. Then, it follows, from the Oka-Cartan Theorem B and Schapira[14], Theorem B42, p.38, that we have

$$H^p(K, \mathcal{A}) \cong \lim \mathrm{ind}_{\tilde{\Omega}\cap M=\Omega} H^p(\tilde{\Omega}, \mathcal{O}) = 0,\ (p > 0),$$

where $\tilde{\Omega}$ runs through all Stein open neighborhoods of Ω with $\tilde{\Omega} \cap M = \Omega$. Q.E.D.

If Ω is an open set in X, we set $\mathcal{O}(\Omega) = \Gamma(\Omega, \mathcal{O})$, the section module on Ω. This space has an FS-space topology with seminorms $p_K(f) = \sup_K |f|$, where K runs over the family of compact subsets of Ω. It is known that $\mathcal{O}(\Omega)$ is a nuclear Fréchet space. Let K be a compact subset of X. We put

$$\mathcal{O}(K) = \lim \mathrm{ind}_{\Omega \supset K} \mathcal{O}(\Omega).$$

$\mathcal{O}(K)$ is the space of holomorphic functions in a neighborhood of K endowed with the inductive limit topology of $\mathcal{O}(\Omega)$ where Ω runs over the family of all complex open neighborhoods of K. It is a nuclear DFS-space(in particular, it is Hausdorff) and its dual $\mathcal{O}'(K)$ is a nuclear FS-space.

Further, every bounded subset of $\mathcal{O}(K)$ is contained and bounded in a space $\mathcal{O}(\Omega)$ with $K \subset \Omega$ (cf. Martineau[12] or Komatsu[11]).

If K is a compact subset of M, we have an isomorphism $\mathcal{A}(K) \cong \mathcal{O}(K)$, where $\mathcal{A}(K)$ denotes the space of real-analytic functions in a neighborhood of K in M. $\mathcal{A}(K)$ is endowed with the topology of $\mathcal{O}(K)$. Then $\mathcal{O}(X)$ is dense in $\mathcal{A}(K)$ by virtue of the embedding theorem(Grauert[3]).

If Ω is an open set in M, let $\mathcal{A}(\Omega)$ be the space of real-analytic functions on Ω equipped with the topology

$$\mathcal{A}(\Omega) = \lim \mathrm{proj}_{K \subset \Omega} \mathcal{A}(K).$$

Here K runs over the family of all compact subsets of Ω. Then $\mathcal{A}(\Omega)$ is a nuclear FS-space.

Now we have the following.

Proposition 1.3. *Let* M_i *be an* n_i*-dimensional and real-analytic manifold countable at infinity and* X_i *a complexification of* $M_i (i = 1, 2)$. *Then we have the following canonical isomorphisms:*

(1) $\mathcal{O}(\Omega_1)\hat{\otimes}\mathcal{O}(\Omega_2) \cong \mathcal{O}(\Omega_1 \times \Omega_2)$, ($\Omega_i$ *is an open set in* $X_i (i = 1, 2)$).

(2) $\mathcal{O}(K_1)\hat{\otimes}\mathcal{O}(K_2) \cong \mathcal{O}(K_1 \times K_2)$, ($K_i$ *is a compact set in* $X_i (i = 1, 2)$).

(3) $\mathcal{A}(K_1)\hat{\otimes}\mathcal{A}(K_2) \cong \mathcal{A}(K_1 \times K_2)$, ($K_i$ *is a compact set in* $M_i (i = 1, 2)$).

(4) $\mathcal{A}(\Omega_1)\hat{\otimes}\mathcal{A}(\Omega_2) \cong \mathcal{A}(\Omega_1 \times \Omega_2)$, ($\Omega_i$ *is an open set in* $M_i (i = 1, 2)$).

Proof. See Ito[7], Proposition, p.30. Q.E.D.

Proposition 1.4. *Let* K_1 *and* K_2 *be two compact sets in* M. *Then the sequence*

$$0 \longrightarrow \mathcal{A}(K_1 \cup K_2) \longrightarrow \mathcal{A}(K_1) \oplus \mathcal{A}(K_2) \longrightarrow \mathcal{A}(K_1 \cap K_2) \longrightarrow 0$$

$$(f_1,\ f_2) \longmapsto f_1 - f_2$$

is exact.

Proof. By virtue of the Mayer-Vietoris Theorem, we have the long exact sequence of cohomology groups

$$0 \longrightarrow \mathcal{A}(K_1 \cup K_2) \longrightarrow \mathcal{A}(K_1) \oplus \mathcal{A}(K_2) \longrightarrow \mathcal{A}(K_1 \cap K_2)$$

$$\longrightarrow H^1(K_1 \cup K_2,\ \mathcal{A}) \longrightarrow \cdots.$$

Then, by the Malgrange Theorem, we have $H^1(K_1 \cup K_2,\ \mathcal{A}) = 0$. Thus we have the conclusion. Q.E.D.

2. Analytic-linear mappings

In this section we recall the notion of analytic-linear mappings following Ito[8].

In the sequel of this article, E is always assumed to be an arbitrary, locally convex, Hausdorff and topological vector space over the complex number field (LCV for short) as far as the contrary is not mentioned.

Definition 2.1. Let Ω be an open set in X and $\mathcal{O}'(\Omega;\ E) = L(\mathcal{O}(\Omega),\ E)(= L_b(\mathcal{O}(\Omega),\ E))$ the space of all continuous linear mappings of $\mathcal{O}(\Omega)$ into E equipped with the topology of uniform convergence on every bounded set in $\mathcal{O}(\Omega)$. We call an element of $\mathcal{O}'(\Omega;\ E)$ an analytic-linear mapping on Ω valued in E or an (E-valued) analytic-linear mapping on Ω. We say that $u \in \mathcal{O}'(\Omega;\ E)$ is carried by a compact subset K of Ω if u can be extended to $\mathcal{O}(K)$. We then call K a carrier of u. We also say that $u \in \mathcal{O}'(\Omega;\ E)$ is carried by an open subset ω of Ω if u is carried by some compact subset of ω. Then ω is said to be a carrier of u. The spaces $\mathcal{O}'(K;\ E) = L(\mathcal{O}(K),\ E), \mathcal{A}'(K;\ E) = L(\mathcal{A}(K),\ E)$ and $\mathcal{A}'(\Omega;\ E) = L(\mathcal{A}(\Omega),\ E)$ are defined in a similar way. We also say their elements to be analytic-linear mappings and define the notion of their carriers in a similar way.

Proposition 2.2. *Let* E *be complete. Then we have the following isomorphisms:*

(1) $\mathcal{O}'(\Omega;\ E) \cong \mathcal{O}'(\Omega)\hat{\otimes}E$, ($\Omega$ *is an open set in* X).

(2) $\mathcal{O}'(K;\ E) \cong \mathcal{O}'(K)\hat{\otimes}E$, ($K$ *is a compact set in* X).

(3) $\mathcal{A}'(K;\ E) \cong \mathcal{A}'(K)\hat{\otimes}E$, ($K$ *is a compact set in* M).

(4) $\mathcal{A}'(\Omega;\ E) \cong \mathcal{A}'(\Omega)\hat{\otimes}E$, *($\Omega$ is an open set in M).*

Proof. See Trèves[15], Proposition 50.5, p.522. Q.E.D.

Definition 2.3. Let K be a compact subset of X. Then we say for K to have the Runge property if all bounded subset of $\mathcal{O}(K)$ is in the closure of a bounded subset, in $\mathcal{O}(K)$, of elements of $\mathcal{O}(X)$.

Proposition 2.4. *Let K be a compact subset of X with the Runge property. Let $u \in \mathcal{O}'(X;\ E)$. Then u is carriable by K if and only if it is carriable by all open neighborhood of K.*

Proof. See Ito[8], Proposition 2.14, p.35. Q.E.D.

The elements of $\mathcal{A}'(M;\ E)$ are called real and analytic-linear mappings. They are analytic-linear mappings on X which are carried by a real compact set in M.

Theorem 2.5. *For an arbitrary family $\{K_i\}_{i\in I}$ of at most countable compact sets in M, we have $\cap_{i\in I}\mathcal{A}'(K_i;\ E) \cong \mathcal{A}'(\cap_{i\in I}K_i;\ E)$.*

Proof. (i) At first we prove the case $I = \{1,2\}$. By virtue of Proposition 2.4, we have the exact sequence

$$0 \longrightarrow \mathcal{A}(K_1 \cup K_2) \longrightarrow \mathcal{A}(K_1) \oplus \mathcal{A}(K_2) \longrightarrow \mathcal{A}(K_1 \cap K_2) \longrightarrow 0.$$

Thus we have the exact sequence

$$0 \longrightarrow \mathcal{A}'(K_1 \cap K_2;\ E) \longrightarrow \mathcal{A}'(K_1;\ E) \oplus \mathcal{A}'(K_2;\ E) \stackrel{\lambda}{\longrightarrow} \mathcal{A}'(K_1 \cup K_2;\ E).$$

Thus we have $\mathrm{Ker}(\lambda) \cong \mathcal{A}'(K_1;\ E) \cap \mathcal{A}'(K_2;\ E) \cong \mathcal{A}'(K_1 \cap K_2;\ E)$.

(ii) Next we prove the case $I = \{1, 2, \cdots, m\}$. We use the induction. The case $m = 2$ holds good by (i). Assuming that the case $m-1$ holds good, we prove the case m. Then we have

$$\begin{aligned} \cap_{i=1}^{m}\mathcal{A}'(K_i;\ E) &= \{\cap_{i=1}^{m-1}\mathcal{A}'(K_i;\ E)\} \cap \mathcal{A}'(K_m;\ E) \\ &\cong \mathcal{A}'(\cap_{i=1}^{m-1}K_i;\ E) \cap \mathcal{A}'(K_m;\ E) \\ &\cong \mathcal{A}'(\{\cap_{i=1}^{m-1}K_i\} \cap K_m;\ E) = \mathcal{A}'(\cap_{i=1}^{m}K_i;\ E). \end{aligned}$$

Thus we have the conclusion in this case.

(iii) In the case $I = \{1,2,3,\cdots\}$. Put $K = \cap_{i\in I}K_i$ and $L_m = \cap_{i=1}^{m}K_i$. Then we have $L_1 \supset L_2 \supset \cdots \supset L_m \supset \cdots \supset K$ and $K = \cap_{m=1}^{\infty}L_m$. Thus we have algebraic isomorphisms

$$\begin{aligned} \mathcal{A}'(K;\ E) &\cong L(\lim\mathrm{ind}_m\mathcal{A}(L_m);\ E) \cong \lim\mathrm{proj}_m\mathcal{A}'(L_m\ E) \\ &\cong \lim\mathrm{proj}_m \cap_{i=1}^{m} \mathcal{A}'(K_i;\ E) \cong \cap_{i=1}^{\infty}\mathcal{A}'(K_i;\ E). \end{aligned}$$

Thus we have the conclusion. Q.E.D.

In the proof of Theorem 2.5, we have used the following.

Lemma 2.6. *Let $\{E_m, u_{mn}(m \le n)\}$ be an inductive system of LCV's and continuous linear mappings and F an arbitrary LCV. Then the family $\{E'_m, u'_{mn}\}$ with $E'_m = L(E_m,\ F)$ and the adjoint mapping $u'_{mn} : E'_n \longrightarrow E'_m$, $(m \le n)$*

becomes a projective system of LCV's and continuous linear mappings u'_{mn} *and we have the algebraic isomorphism*

$$L(\lim \operatorname{ind}_m E_m,\ F) \cong \lim \operatorname{proj}_m L(E_m,\ F).$$

Proof. The family $\{E'_m, u'_{mn}\}$ is evidently a projective system. Thus we have to prove the algebraic isomorphism

$$L(\lim \operatorname{ind}_m E_m,\ F) \cong \lim \operatorname{proj}_m L(E_m,\ F).$$

Put $E = \lim \operatorname{ind}_m E_m$ and $u_m : E_m \longrightarrow E$ be the canonical map. The adjoint map u'_m of u_m defines the linear mapping

$$\phi : L(E,\ F) \longrightarrow \lim \operatorname{proj}_m L(E_m,\ F),$$

$$T \longrightarrow \{u'_m T\}.$$

In fact, we have $u'_{mn}(u'_n T) = (u_n \circ u_{mn})' T = u'_m T$. Then ϕ is injective. In fact, let $\phi T = \{u'_m T\} = 0$, i.e. $u'_m T = 0$ for all m. Then, for $x \in E$ and $x_m \in E_m$ with $u_m x_m = x$, $T(x) = T(u_m x_m) = (u'_m T)(x_m) = 0$ holds. Thus we have $T = 0$. Now let $\{T_m\} \in \lim \operatorname{proj}_m L(E_m,\ F)$ be given. Then $T_m = u'_{mn} T_n = T_n \circ u_{mn}$, $(m \leq n)$ holds good. Thus there exists $T \in L(\lim \operatorname{ind}_m E_m,\ F)$ with $T_m = T \circ u_m = u'_m T$. This means that $\phi T = \{u'_m T\} = \{T_m\}$. Thus ϕ is an algebraic isomorphism. Q.E.D.

Theorem 2.7. *Let* $u \in \mathcal{A}'(M;\ E)$ *with* $u \neq 0$. *Then there exists the smallest compact set in* M *which carries* u. *We call it the support of* u *and denote it by* supp(u).

Proof. Among all carriers of u, we have only to consider compact carriers of u. Let $\{K_\alpha\}$ be the family of all compact carriers of u in M. Then let $\{K_\beta\}$ be an arbitrary totally ordered subfamily of $\{K_\alpha\}$ with $K_\beta \supset K_{\beta'}$ $(\beta \leq \beta')$. Then we can choose a subsequence $\{K_j\}$ of $\{K_\beta\}$ with $\cap_j K_j = \cap_\beta K_\beta = K$. In fact, let $\{U_j\}$ be a countable family of open neighborhoods of K with $U_1 \supset U_2 \supset \cdots$ and $\cap_j U_j = K$. Then we choose K_j as the largest compact set among the subfamily of all compact sets $K_\beta \in \{K_\beta\}$ contained in U_j. Then the projective system $\{\mathcal{A}'(K_j;\ E)\}$ is cofinal with the projective system $\{\mathcal{A}'(K_\beta;\ E)\}$. Thus, by virtue of Theorem 2.5, we have the algebraic isomorphisms

$$\begin{aligned} \cap_\beta \mathcal{A}'(K_\beta;\ E) &\cong \lim \operatorname{proj}_\beta \mathcal{A}'(K_\beta;\ E) \\ &\cong \lim \operatorname{proj}_j \mathcal{A}'(K_j;\ E) \cong \mathcal{A}'(K;\ E). \end{aligned}$$

Then $u \in \mathcal{A}'(K;\ E)$. Thus K is a minimal compact carrier of u. Thus, by virtue of Zorn's Lemma, we have the conclusion. Q.E.D.

For u, u_1, $u_2 \in \mathcal{A}'(M;\ E)$, we have

$$\operatorname{supp}(u_1 + u_2) \subset \operatorname{supp}(u_1) \cup \operatorname{supp}(u_2),$$

$$\mathrm{supp}(\lambda u) \subset \mathrm{supp}(u), \quad (\lambda \in \boldsymbol{C}).$$

Proposition 2.8. *For every two compact sets K_1 and K_2 in M with $K_1 \subset K_2$, there exists a continuous injection $i_{K_1,K_2} : \mathcal{A}'(K_1;\, E) \longrightarrow \mathcal{A}'(K_2;\, E)$.*

Proof. (1) Let ${}^t i_{K_1,K_2} : \mathcal{A}(K_2) \longrightarrow \mathcal{A}(K_1)$ be a canonical mapping. Then ${}^t i_{K_1,K_2}$ is evidently continuous. Let $i_{K_1,K_2} : \mathcal{A}'(K_1;\, E) \longrightarrow \mathcal{A}'(K_2;\, E)$ be its adjoint map. Then i_{K_1,K_2} is evidently a continuous injection. Q.E.D.

Proposition 2.9. *Let K_1 and K_2 be two compact sets in M with $K_1 \subset K_2$. Further, assume that every connected component of K_2 intersects K_1. Then i_{K_1,K_2} has the dense range in $\mathcal{A}'(K_2;\, E)$.*

Proof. By the assumptions, the canonical mapping ${}^t i_{K_1,K_2}$ is injective. Thus $\mathcal{A}'(K_1)$ is dense in $\mathcal{A}'(K_2)$. Then we have the inclusions,

$$\begin{aligned} \mathcal{A}'(K_1) \otimes E &\hookrightarrow L(\mathcal{A}(K_1),\, E) \hookrightarrow L(\mathcal{A}(K_2),\, E) \\ &\hookrightarrow L(\mathcal{A}(K_2),\, \hat{E}) = \mathcal{A}'(K_2;\, \hat{E}). \end{aligned}$$

Here $\hat{E}$ denotes the completion of E. Since $\mathcal{A}'(K_1) \otimes E$ is dense in $\mathcal{A}'(K_2;\, \hat{E})$, we have the conclusion. Q.E.D.

Let now Ω be an open subset of M and K a compact subset of Ω. We call the "envelope" of K (in Ω) and denote by $\tilde{K}$, the union of K and all the relatively compact connected components (in Ω) of $\Omega \setminus K$. It is again a compact set.

Corollary 2.10. *Let Ω be a relatively compact subset of M and K_1, K_2 ($K_1 \subset K_2$) two compact subsets of Ω such that $K_i = \tilde{K}_i$ holds ($i = 1, 2$). Then $\mathcal{A}'((\Omega \setminus K_2)^{\mathrm{cl}};\, E)$ is dense in $\mathcal{A}'((\Omega \setminus K_1)^{\mathrm{cl}};\, E)$.*

3. Vector-valued Sato hyperfunctions

Let E be an LCV. First we consider vector-valued Sato hyperfunctions on a relatively compact open set in M.

Let Ω be a relatively compact open subset of M. We put

$$\mathcal{B}(\Omega;\, E) = \mathcal{A}'(\Omega^{\mathrm{cl}};\, E)/\mathcal{A}'(\partial\Omega;\, E).$$

Then, since $\mathcal{A}'(\partial\Omega;\, E)$ is dense in $\mathcal{A}'(\Omega^{\mathrm{cl}};\, E)$, $\mathcal{B}(\Omega;\, E)$ is not endowed with any nontrivial topology.

Definition 3.1. An element of $\mathcal{B}(\Omega;\, E)$ is called a Sato hyperfunction on ω valued in E or an E-valued Sato hyperfunction on ω.

Let K be a compact set in M containing Ω. Then, by virtue of Proposition 2.8, we have the canonical map

$$\mathcal{A}'(\Omega^{\mathrm{cl}};\, E) \longrightarrow \mathcal{A}'(K;\, E) \longrightarrow \mathcal{A}'(K;\, E)/\mathcal{A}'(K \setminus \Omega;\, E),$$

whose kernel is the space

$$\mathcal{A}'(\Omega^{\mathrm{cl}};\, E) \cap \mathcal{A}'(K \setminus \Omega;\, E) \cong \mathcal{A}'(\partial\Omega;\, E).$$

Thus we have the isomorphism

$$\mathcal{B}(\Omega;\ E) = \mathcal{A}'(\Omega^{\mathrm{cl}};\ E)/\mathcal{A}'(\partial\Omega;\ E) \cong \mathcal{A}'(K;\ E)/\mathcal{A}'(K \setminus \Omega;\ E).$$

Let now ω be an open subset of Ω. Then the mapping

$$\mathcal{A}'(\Omega^{\mathrm{cl}};\ E) \longrightarrow \mathcal{A}'(\Omega^{\mathrm{cl}};\ E)/\mathcal{A}'(\Omega^{\mathrm{cl}} \setminus \omega;\ E)$$

defines a mapping

$$\mathcal{B}(\Omega;\ E) \longrightarrow \mathcal{B}(\omega :\ E),$$

because $\mathcal{A}'(\partial\Omega;\ E) \subset \mathcal{A}'(\Omega^{\mathrm{cl}} \setminus \omega;\ E)$ holds. This mapping is called the restriction.

If $T \in \mathcal{B}(\Omega;\ E)$, we denote by $T|_\omega$ its image in $\mathcal{B}(\omega;\ E)$. It is clear that, if $\Omega_3 \subset \Omega_2 \subset \Omega_1$ and $T \in \mathcal{B}(\Omega_1;\ E)$, we have

$$(T|_{\Omega_2})|_{\Omega_3} = T|_{\Omega_3}.$$

Thus we have the following.

Proposition 3.2. *Let Ω be a relatively compact open set in M. Then the collection $\{B(\omega;\ E); \omega$ is an open subset of $\Omega\}$ becomes a presheaf (of vector spaces) over Ω.*

Proposition 3.3. *We use the notation in Proposition 3.2. Let $\omega = \cup_{i\in I}\omega_i$ be a union of open subsets ω_i of $\Omega(i \in I)$ and $T \in \mathcal{B}(\omega;\ E)$ with $T|_{\omega_i} = 0$ for all $i \in I$. Then we have $T = 0$.*

Proof. By the assumptions, if $u_T \in \mathcal{A}'(\omega^{\mathrm{cl}};\ E)$ is a representative of T, the image of u_T in $\mathcal{A}'(\omega^{\mathrm{cl}};\ E)/\mathcal{A}'(\omega^{\mathrm{cl}} \setminus \omega_i;\ E)$ is zero for all $i \in I$. From here we have

$$u_T \in \mathcal{A}'(\omega^{\mathrm{cl}} \setminus \omega_i;\ E) \text{ for all } i \in I.$$

Namely, we have

$$u_T \in \cap_{i\in I}\mathcal{A}'(\omega^{\mathrm{cl}} \setminus \omega_i;\ E).$$

Here we have

$$\begin{aligned}&\cap_{i\in I}\mathcal{A}'(\omega^{\mathrm{cl}} \setminus \omega_i;\ E) \cong \mathcal{A}'(\cap_{i\in I}(\omega^{\mathrm{cl}} \setminus \omega_i);\ E)\\ &= \mathcal{A}'(\omega^{\mathrm{cl}} \setminus \cup_{i\in I}\omega_i;\ E) = \mathcal{A}'(\omega^{\mathrm{cl}} \setminus \omega;\ E) = \mathcal{A}'(\partial\omega;\ E).\end{aligned}$$

Hence we have $\mathrm{supp}(u_T) \subset \partial\omega$, so that $T = 0$ holds. Q.E.D.

Thus we have seen that the presheaf $\{\mathcal{B}(\omega :\ E); \omega$ is an open subset of $\Omega\}$ satisfies the condition (S1) of Bredon[1], p.5. But if E is not a Fréchet space, this presheaf does not satisfy the condition (S2) of Bredon[1], p.6. Thus this presheaf does not become a sheaf unless E is a Fréchet space.

Proposition 3.4. *We use the notation in Proposition 3.2. If ω is an open subset of Ω and $T \in \mathcal{B}(\omega;\ E)$, then there exists $\tilde{T} \in \mathcal{B}(\Omega;\ E)$ such that $\tilde{T}|_\omega = T$.*

Proof. Let $u_T \in \mathcal{A}'(\omega^{\mathrm{cl}};\ E)$ be a representative of T. Then we have $u_T \in \mathcal{A}'(\Omega^{\mathrm{cl}};\ E)$. Thus we define $\tilde{T} \in \mathcal{B}(\Omega;\ E)$ to be the image of u_T in $\mathcal{B}(\Omega;\ E)$. Then evidently we have $\tilde{T}|_\omega = T$. Q.E.D.

By virtue of Proposition 3.3, we can define the support of $T \in \mathcal{B}(\Omega; E)$. Namely, a compact set K in Ω is said to be the support of T if $\Omega \setminus K$ is the largest open set in Ω for which $T|_{\Omega\setminus K} = 0$ holds. Then we denote $K = \mathrm{supp}(T)$.

Proposition 3.5. *We use the notation in Proposition 3.2. Let K be a compact subset of Ω and put*

$$\mathcal{B}_K(\Omega;\ E) = \{T \in \mathcal{B}(\Omega;\ E); \mathrm{supp}(T) \subset K\}.$$

Then we have the inclusion

$$\mathcal{A}'(K;\ E) \subset \mathcal{B}_K(\Omega;\ E).$$

Proof. By the assumptions, we have the inclusion map

$$\mathcal{A}'(K;\ E) \longrightarrow \mathcal{A}'(\Omega^{\mathrm{cl}};\ E)/\mathcal{A}'(\partial\Omega;\ E) = \mathcal{B}(\Omega;\ E).$$

Let $u \in \mathcal{A}'(K; E)$ and $[u]$ its image in $\mathcal{B}(\Omega; E)$. We consider the restriction $[u]|_{\Omega\setminus K}$. Since $\mathcal{B}(\Omega\setminus K;\ E) \cong \mathcal{A}'(\Omega^{\mathrm{cl}};\ E)/\mathcal{A}'(\Omega^{\mathrm{cl}}\setminus(\Omega\setminus K);\ E) = \mathcal{A}'(\Omega^{\mathrm{cl}};\ E)/\mathcal{A}'(\partial\Omega\cup K;\ E)$ holds and $u \in \mathcal{A}'(K;\ E) \subset \mathcal{A}'(\partial\Omega\cup K;\ E)$ holds, we have $[u]|_{\Omega\setminus K} = 0$. Thus we have $\mathcal{A}'(K;\ E) \subset \mathcal{B}_K(\Omega;\ E)$. Q.E.D.

In order to prove the inclusion $\mathcal{B}_K(\Omega;\ E) \subset \mathcal{A}'(K;\ E)$, it is sufficient to know the following.

Problem A. For two compact sets K_1 and K_2 in M and put $K = K_1 \cup K_2$. Then is the sequence

$$\mathcal{A}'(K_1;\ E) \oplus \mathcal{A}'(K_2;\ E) \longrightarrow \mathcal{A}'(K;\ E) \longrightarrow 0$$

exact?

If E is a Fréchet space, the answer to the Problem A is affirmative(cf. Ito[7], Proposition 2.3, p.33). But in general we do not know any answer.

Next we consider E-valued Sato hyperfunctions on M.

Let $\{\mathcal{B}_1(\Omega;\ E); \Omega$ is an open set in $M\}$ be the presheaf over M defined as follows:

If Ω is not relatively compact, $\mathcal{B}_1(\Omega;\ E) = \{0\}$.

If Ω is relatively compact, $\mathcal{B}_1(\Omega;\ E) = \mathcal{B}(\Omega;\ E)$.

Then restrictions are defined as follows:

$$\mathcal{B}_1(\Omega;\ E) \longrightarrow \mathcal{B}_1(\omega;\ E)$$

$$0 \longrightarrow 0 \text{ if } \Omega \text{ is not relatively compact with } \Omega \supset \omega,$$

$$T \longrightarrow T|_\omega \text{ if } \Omega \text{ is relatively compact with } \Omega \supset \omega.$$

This presheaf satisfies the condition (S1) of sheaves but not (S2) (cf. Bredon[1], pp.5-6).

We denote by ${}^E\mathcal{B}$ the sheaf associated to this presheaf $\{\mathcal{B}_1(\Omega;\ E); \Omega$ is an open set in $M\}$. It is a sheaf of vector spaces over M.

Definition 3.6. The sheaf ${}^E\mathcal{B}$ is called the sheaf of E-valued Sato hyperfunctions over M.

Then if Ω is an open set in M and $T \in \Gamma(\Omega,\ {}^E\mathcal{B}) = \mathcal{B}(\Omega;\ E)$, T is defined to be an equivalence class $\{(\Omega_i, T_i)_{i\in I}\}$ as follows:

The family $(\Omega_i, T_i)_{i\in I}$ of open sets Ω_i in Ω and $T_i \in \mathcal{B}(\Omega_i;\ E)$, $(i \in I)$ is determined as follows:

$(\Omega_i)_{i\in I}$ is a covering of Ω and Ω_i's are relatively compact open sets and $T_i \in \mathcal{B}(\Omega_i;\ E)$ satisfies

$$T_i|_{\Omega_i\cap\Omega_j} = T_j|_{\Omega_i\cap\Omega_j}.$$

Two such families $(\Omega_i, T_i)_{i\in I}$ and $(\Omega_{i'}, T_{i'})_{i'\in I'}$ are defined to be equivalent if

$$T_i|_{\Omega_i\cap\Omega_{i'}} = T_{i'}|_{\Omega_i\cap\Omega_{i'}},\ (i \in I,\ i' \in I')$$

holds. This relation is in fact an equivalence relation by virtue of (S1). Then $\Gamma(\Omega,\ {}^E\mathcal{B}) = \mathcal{B}(\Omega,\ E)$ is defined to be the quotient space of the space of all families $\{(\Omega_i,\ T_i)_{i\in I}\}$ of the above type with respect to this equivalence relation. Then $T = (\Omega_i,\ T_i)_{i\in I}$ is defined so that $T|_{\Omega_i} = T_i$, $(i \in I)$ holds.

The symbol $\mathcal{B}(\Omega,\ E) = \Gamma(\Omega,\ {}^E\mathcal{B})$ is used as the section module of the sheaf ${}^E\mathcal{B}$ in abuse of languages. The symbol $\mathcal{B}(\Omega,\ E) = \mathcal{A}'(\Omega^{\mathrm{cl}};\ E)/\mathcal{A}'(\partial\Omega;\ E)$ for relatively compact open set Ω and the symbol $\mathcal{B}(\Omega,\ E) = \Gamma(\Omega,\ {}^E\mathcal{B})$ are distinguished in the context.

Here we present the following problem.

Problem B. Is the sheaf ${}^E\mathcal{B}$ flabby?

If E is a Fréchet space, the answer to the Problem B is affirmative(cf. Ito[7], Theorem 4.1, p.41). But in general we do not know any answer. If the presheaf $\{\mathcal{B}(\omega,\ E) = \mathcal{A}'(\omega^{\mathrm{cl}};\ E)/\mathcal{A}'(\partial\omega;\ E); \omega$ is an open subset of $\Omega\}$ over a relatively compact open set Ω in M becomes a flabby sheaf, then we can show that the answer to the Problem B is affirmative by a similar way to Ito[10], Lemma 1.2.5, p.221.

4. Operations on analytic-linear mappings and Kernel Theorems

In this section we now define several operations on analytic-linear mappings.

4.1. Tensor products and Kernel Theorems. At first we recall the tensor product of analytic functionals.

Proposition 4.1. *Let M_i be an n_i-dimensional, real-analytic manifold countable at infinity and X_i its complexification $(i = 1, 2)$. Then we have the following canonical isomorphisms:*

(1) $\mathcal{O}'(\Omega_1)\hat{\otimes}\mathcal{O}'(\Omega_2) \cong L(\mathcal{O}(\Omega_1),\ \mathcal{O}'(\Omega_2)) \cong \mathcal{O}'(\Omega_1 \times \Omega_2)$,
(Ω_i *is an open set in* $X_i (i = 1, 2)$).

(2) $\mathcal{O}'(K_1)\hat{\otimes}\mathcal{O}'(K_2) \cong L(\mathcal{O}(K_1), \mathcal{O}'(K_2)) \cong \mathcal{O}'(K_1 \times K_2)$,
(K_i is a compact set in $X_i (i = 1, 2)$).

(3) $\mathcal{A}'(K_1)\hat{\otimes}\mathcal{A}'(K_2) \cong L(\mathcal{A}(K_1), \mathcal{A}'(K_2)) \cong \mathcal{A}'(K_1 \times K_2)$,
(K_i is a compact set in $M_i (i = 1, 2)$).

(4) $\mathcal{A}'(\Omega_1)\hat{\otimes}\mathcal{A}'(\Omega_2) \cong L(\mathcal{A}(\Omega_1), \mathcal{A}'(\Omega_2)) \cong \mathcal{A}'(\Omega_1 \times \Omega_2)$,
(Ω_i is an open set in $M_i (i = 1, 2)$).

Proof. See Ito[7], Proposition 3.1, p.35. Q.E.D.

We note that Proposition 4.1 establishes analogs of Schwartz's Kernel Theorem in the case of analytic functionals.

Next we consider tensor products of analytic-linear mappings. In this section, we assume that E_1 and E_2 are two complete LCV's and put $E = E_1\hat{\otimes}_\omega E_2$, where ω stands for the ϵ- or π-topology in the sense of Trèves[15].

Theorem 4.2. *Let M_i and X_i be as in Proposition 4.1, and E_1 and E_2 two complete LCV's. Put $E = E_1\hat{\otimes}_\omega E_2$, where ω stands for the ϵ- or π-topology. Then we have the following canonical isomorphisms:*

(1) $\mathcal{O}'(\Omega_1; E_1)\hat{\otimes}_\omega\mathcal{O}'(\Omega_2; E_2) \cong \mathcal{O}'(\Omega_1 \times \Omega_2; E)$,
(Ω_i is an open set in $X_i (i = 1, 2)$).

(2) $\mathcal{O}'(K_1; E_1)\hat{\otimes}_\omega\mathcal{O}'(K_2; E_2) \cong \mathcal{O}'(K_1 \times K_2; E)$,
(K_i is a compact set in $X_i (i = 1, 2)$).

(3) $\mathcal{A}'(K_1; E_1)\hat{\otimes}_\omega\mathcal{A}'(K_2; E_2) \cong \mathcal{A}'(K_1 \times K_2; E)$,
(K_i is a compact set in $M_i (i = 1, 2)$).

(4) $\mathcal{A}'(\Omega_1; E_1)\hat{\otimes}_\omega\mathcal{A}'(\Omega_2; E_2) \cong \mathcal{A}'(\Omega_1 \times \Omega_2; E)$,
(Ω_i is an open set in $M_i (i = 1, 2)$).

Proof. It goes in a similar way to Ito[7], Proposition 3.2, p.36. Q.E.D.

We note that Theorem 4.2 establishes analogs of Schwartz's Kernel Theorem in the case of analytic-linear mappings.

With the help of Theorem 4.2, we have the following definitions of tensor products of analytic-linear mappings.

Definition 4.3. We use the notation in Theorem 4.2. Let $u_1 = \varphi_1 \otimes e_1 \in \mathcal{O}'(\Omega_1; E_1)$ and $u_2 = \varphi_2 \otimes e_2 \in \mathcal{O}'(\Omega_2; E_2)$, where $\varphi_1 \in \mathcal{O}'(\Omega_1)$ and $\varphi_2 \in \mathcal{O}'(\Omega_2)$ and $e_i \in E_i (i = 1, 2)$. Then we define $u_1 \otimes_\omega u_2$ by the following relation:

$$u_1 \otimes_\omega u_2 = (\varphi_1 \otimes \varphi_2) \otimes (e_1 \otimes_\omega e_2)$$

i.e.,

$$(u_1 \otimes_\omega u_2)(f_1 \otimes f_2) = \varphi_1(f_1)\varphi_2(f_2)(e_1 \otimes_\omega e_2),$$
$$\text{for } f_i \in \mathcal{O}(\Omega_i), \ (i = 1, 2).$$

In all other cases, we define tensor products of analytic-linear mappings of each type similarly.

In all real cases, we have

$$\mathrm{supp}(u_1 \otimes_\omega u_2) \subset \mathrm{supp}(u_1) \times \mathrm{supp}(u_2).$$

4.2. Multiplication by a holomorphic or a real-analytic function. Let Ω be an open set in X. For $f \in \mathcal{O}(\Omega)$ and $u \in \mathcal{O}'(\Omega;\, E)$ we define $fu \in \mathcal{O}'(\Omega;\, E)$ by the formula

$$(fu)(g) = u(fg) \text{ for all } g \in \mathcal{O}(\Omega).$$

By this definition $\mathcal{O}'(\Omega;\, E)$ is an $\mathcal{O}(\Omega)$-module.

For a compact set K in X(or M) and an open set Ω in M, we can define an $\mathcal{O}(K)$-(resp. $\mathcal{A}(K)$-, resp. $\mathcal{A}(\Omega)$-) module structure of $\mathcal{O}'(\Omega;\, E)$ (resp. $\mathcal{A}'(K;\, E)$, resp. $\mathcal{A}'(\Omega;\, E)$) in a similar way.

For a real and analytic-linear mapping u and a real-analytic function f, we have

$$\mathrm{supp}(fu) \subset \mathrm{supp}(u).$$

4.3. Differentiation. Let X be an analytic manifold and Ω an open set in X.

Let P be a differential operator on Ω of finite order with coefficients in $\mathcal{O}(\Omega)$ and P^* a formal adjoint differential operator. Then, for an arbitrary analytic-linear mapping $u \in \mathcal{O}'(\Omega;\, E)$ on the open set Ω, we define Pu by the formula

$$(Pu)(f) = u(P^*f), \text{ for every } f \in \mathcal{O}(\Omega).$$

In a similar way, we can define the derivative of analytic-linear mappings of each type.

4.4. Analytic diffeomorphisms. If Ω_1 and Ω_2 are two open sets in X, $w = \Phi(z)$ denotes a complex-analytic diffeomorphism of Ω_1 onto Ω_2.

Then, for $u \in \mathcal{O}'(\Omega_2;\, E)$, we define $\Phi^*u \in \mathcal{O}'(\Omega_1;\, E)$ by the formula

$$(\Phi^*u)(f) = u((f \circ \Phi^{-1})|J|), \text{ for } f \in \mathcal{O}(\Omega_1),$$

where $|J|$ is the absolute value of the Jacobian J of the mapping Φ^{-1}.

If K_1 and K_2 are two compact sets in X and if $w = \Phi(z)$ is a complex-analytic diffeomorphism which maps a certain open neighborhood in X of K_1 onto a certain open neighborhood in X of K_2 such that $\Phi(K_1) = K_2$, then, for $u \in \mathcal{O}'(K_2;\, E)$, we define $\Phi^*u \in \mathcal{O}'(K_1;\, E)$ by the formula

$$(\Phi^*u)(f) = u((f \circ \Phi^{-1})|J|), \text{ for } f \in \mathcal{O}(K_1),$$

If K_1 and K_2 are two compact sets in M, this is a special case of the above. But, in this case, we have

$$\mathrm{supp}(\Phi^*u) = \Phi^{-1}(\mathrm{supp}(u)), \text{ for } u \in \mathcal{A}'(K_2;\, E).$$

At last, if Ω_1 and Ω_2 are two open sets in M and $y = \Phi(x)$ is a real-analytic diffeomorphism of Ω_1 onto Ω_2, then, for $u \in \mathcal{A}'(\Omega_2;\, E)$, we define $\Phi^* u \in \mathcal{A}'(\Omega_1;\, E)$ by the formula

$$(\Phi^* u)(f) = u((f \circ \Phi^{-1})|J|), \text{ for } f \in \mathcal{A}(\Omega_1).$$

Then we have

$$\operatorname{supp}(\Phi^* u) = \Phi^{-1}(\operatorname{supp}(u)), \text{ for } u \in \mathcal{A}'(\Omega_2;\, E).$$

5. Operations on vector-valued Sato hyperfunctions and Kernel Theorems

In this section we define several operations on E-valued Sato hyperfunctions on M.

5.1. Tensor products and Kernel Theorems. In this subsection, we assume that E_1 and E_2 are two complete LCV's and put $E = E_1 \hat{\otimes}_\omega E_2$, where ω stands for the ϵ- or π-topology in the sense of Trèves[15]. Let $\{\mathcal{B}(\Omega;\, E); \Omega$ is an open set in $M\}$ be the presheaf of E-valued Sato hyperfunctions.

Let Ω_i be a relatively compact open set in $M_i (i = 1, 2)$ and $T_i \in \mathcal{B}(\Omega_i,\, E_i)$, $(i = 1, 2)$. Let then $\overline{T}_i \in \mathcal{A}'(\Omega_i^{\mathrm{cl}},\, E_i)$ so that $\overline{T}_i|_{\Omega_i} = T_i$, for $i = 1, 2$. Here $\overline{T}_i|_{\Omega_i}$ denotes the image of $\overline{T}_i$ in $\mathcal{B}(\Omega_i,\, E_i)$. Then we have $\overline{T}_1 \otimes_\omega \overline{T}_2 \in \mathcal{A}'(\Omega_1^{\mathrm{cl}} \times \Omega_2^{\mathrm{cl}};\, E)$. Then we can see that $\overline{T}_1 \otimes_\omega \overline{T}_2|_{\Omega_1 \times \Omega_2}$ does not depend on the choice of representatives $\overline{T}_1$ and $\overline{T}_2$ of T_1 and T_2 respectively. Thus $\overline{T}_1 \otimes_\omega \overline{T}_2|_{\Omega_1 \times \Omega_2}$ is an E-valued Sato hyperfunction in $\mathcal{B}(\Omega_1 \times \Omega_2;\, E)$ which depends only on T_1 and T_2. We denote this by $T_1 \otimes_\omega T_2$ and call it the tensor product of T_1 and T_2. $T_1 \otimes_\omega T_2$ has the properties of tensor products of vectors. Then we have

$$\operatorname{supp}(T_1 \otimes_\omega T_2) \subset \operatorname{supp}(T_1) \times \operatorname{supp}(T_2).$$

Theorem 5.1. *Let E_1 and E_2 be two complete LCV's and put $E = E_1 \hat{\otimes}_\omega E_2$, where ω stands for the ϵ- or π-topology. Let Ω_1 and Ω_2 be two relatively compact open sets in M_1 and M_2 respectively. Then we have the canonical algebraic isomorphism*

$$\mathcal{B}(\Omega_1 \times \Omega_2;\, E) \cong \mathcal{B}(\Omega_1;\, E_1) \otimes_\omega \mathcal{B}(\Omega_2;\, E_2).$$

This is an analog of Schwartz's Kernel Theorem.

5.2. Multiplication by a real-analytic function. Let Ω be a relatively compact open set in M. If $T \in \mathcal{B}(\Omega;\, E)$ and $f \in \mathcal{A}(\Omega^{\mathrm{cl}})$, then we define fT as follows. Let $\overline{T} \in \mathcal{A}'(\Omega^{\mathrm{cl}};\, E)$ such that $\overline{T}|_\Omega = T$ holds. Then $f\overline{T} \in \mathcal{A}(\Omega^{\mathrm{cl}};\, E)$. Then we can see that $f\overline{T}|_\Omega$ does not depend on the choice of the representative $\overline{T}$. Thus $f\overline{T}|_\Omega$ is an E-valued Sato hyperfunction in $\mathcal{B}(\Omega;\, E)$ which depends only on f and T, and which we denote by fT.

5.3 Differentiation. Let Ω be a relatively compact open set in M and P a differential operator of finite order with coefficients in $\mathcal{A}(\Omega^{\mathrm{cl}})$. Let $T \in \mathcal{B}(\Omega; E)$ and $\overline{T} \in \mathcal{A}'(\Omega^{\mathrm{cl}}; E)$ such that $\overline{T}|_\Omega = T$. Then, using the result in the subsection 4.3, we define PT by the formula $PT = P\overline{T}|_\Omega$ which depends only on T.

5.4. Analytic diffeomorphisms. Let Ω_1 and Ω_2 are two relatively compact open sets in M_1 and M_2 respectively and $y = \Phi(x)$ a complex-analytic diffeomorphism which maps a certain complex, open neighborhood of Ω_1^{cl} onto a certain complex, open neighborhood of Ω_2^{cl} such that $\Phi(\Omega_1^{\mathrm{cl}}) = \Omega_2^{\mathrm{cl}}$. Then, for $T \in \mathcal{B}(\Omega_2; E)$, let $\overline{T} \in \mathcal{A}'(\Omega_2^{\mathrm{cl}}; E)$ with $\overline{T}|_{\Omega_2} = T$. Then, using the result in the subsection 4.4, we define $\Phi^*\overline{T} \in \mathcal{A}'(\Omega_1^{\mathrm{cl}}; E)$. Then we can see that $\Phi^*\overline{T}|_{\Omega_1}$ does not depend on the choice of representatives $\overline{T}$. Thus $\Phi^*\overline{T}|_{\Omega_1}$ is an E-valued Sato hyperfunction on Ω_1 which depends only on T. We denote this by Φ^*T. Then we have

$$\mathrm{supp}(\Phi^*T) = \Phi^{-1}(\mathrm{supp}(T)), \text{ for } T \in \mathcal{B}(\Omega_2; E).$$

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