IEICE TRANS. FUNDAMENTALS, VOL. E78–A, NO. 6 JUNE 1995

PAPER

Composite Dynamical System for Controlling Chaos

Tetsushi UETA† *and* **Hiroshi KAWAKAMI**†, *Members*

SUMMARY We propose a stabilization method of unstable periodic orbits embedded in a chaotic attractor of continuous-time system by using discrete state feedback controller. The controller is designed systematically by the Poincaré mapping and its derivatives. Although the output of the controller is applied periodically to system parameter as small perturbations discontinuously, the controlled orbit accomplishes C^0. As the stability of a specific orbit is completely determined by the design of controller, we can also use the method to destabilize a stable periodic orbit. The destabilization method may be effectively applied to escape from a local minimum in various optimization problems. As an example of the stabilization and destabilization, some numerical results of Duffing's equation are illustrated.

key words: *controlling chaos, Poincaré mapping, stabilization, destabilization*

1. Introduction

Recently, the topic about controlling chaos is one of remarkable researches in engineering fields. In 1990, Otto, Grebogi, Yorke [1] proposed a standard method to stabilize an unstable periodic orbit called target embedded in a chaotic attractor. But this method requires the target being a saddle, i.e., the target must possess stable manifolds to determine a feedback gain. Romeiras, Grebogi, Ott, Dayawansa [2] applied a conventional state feedback theory to controlling chaos. By this method any types of unstable fixed or periodic point in a chaotic state can be stabilized by the pole assignment technique.

On the other hand, many other approaches are proposed, e.g., parameter variation technique, absorber, entrainment and feedback method, etc., the outlines of them are given by Ogorzalek [3]. The central techniques using control theory [4], [5] are traditional state feedback control: the feedback signal determined by difference between the target and an orbit is applied to state of the system continuously, thus all information of the target wave form are necessary for stabilization.

In this paper we propose the composite dynamical system as a method for controlling chaos. This system constructed by the original differential equation and difference equation derived from linearization in the neighborhood of the target by the Poincaré mapping. We have only to design a controller stabilizing unstable characteristics of the target on the discrete system described by the difference equation. Some significant features must be pointed out compared with Ref. [2]:

- How to embed the control value to the parameter, which is implicit in many articles, is clarified. Although the parameter varies discontinuously by the control, a controlled orbit can be C^0 since the state space is never manipulated.
- To calculate the controller, we can obtain derivatives of the Poincaré mapping numerically without using analytic or embedding methods.

Consequently, we develop the systematic design method for controlling chaos in case that the mathematical model is given. Moreover the method is applied to destabilize a stable orbit to escape from an undesirable stable state. We show some illustrations of stabilizing or destabilizing the target of the Duffing's equation.

2. System Equation and Its Poincaré Mapping

For simplicity, let us consider an n-dimensional nonautonomous ordinary differential equation (ODE):

$$\frac{d\boldsymbol{x}}{dt} = \boldsymbol{f}(t, \boldsymbol{x}, \boldsymbol{\lambda}) \tag{1}$$

where, $\boldsymbol{x} \in \boldsymbol{R}^n$ is the state vector and $\boldsymbol{\lambda} \in \boldsymbol{R}^r$ is the system parameter. We assume that $\boldsymbol{f}$ is periodic in t with period 2π:

$$\boldsymbol{f}(t + 2\pi, \boldsymbol{x}, \boldsymbol{\lambda}) = \boldsymbol{f}(t, \boldsymbol{x}, \boldsymbol{\lambda}) \tag{2}$$

and sufficiently differentiable for all variables. Suppose also that Eq. (1) have a unique solution for the initial value problem. We denote the solution $\boldsymbol{x}(t)$ with initial value $\boldsymbol{x}_0$ at $t = 0$ as:

$$\boldsymbol{x}(t) = \boldsymbol{\varphi}(t, \boldsymbol{x}_0, \boldsymbol{\lambda}) \tag{3}$$

Thus the relation

$$\boldsymbol{x}(0) = \boldsymbol{\varphi}(0, \boldsymbol{x}_0, \boldsymbol{\lambda}) = \boldsymbol{x}_0 \tag{4}$$

holds. Let us define a differentiable mapping

$$\begin{array}{rcrcl} T : \boldsymbol{R}^n \times \boldsymbol{R}^r & \to & \boldsymbol{R}^n & & \\ (\boldsymbol{x}_0, \boldsymbol{\lambda}) & \mapsto & \boldsymbol{x}_1 & = & T(\boldsymbol{x}_0, \boldsymbol{\lambda}) \\ & & & = & \boldsymbol{\varphi}(2\pi, \boldsymbol{x}_0, \boldsymbol{\lambda}). \end{array} \tag{5}$$

For a fixed $\boldsymbol{\lambda}$ the mapping T becomes the ordinary

Manuscript received November 10, 1994.
Manuscript revised February 13, 1995.

†The authors are with the Faculty of Engineering, The University of Tokushima, Tokushima-shi, 770 Japan.

Poincaré mapping. For the latter use we call the sampling interval 2π the Poincaré sampling period or simply Poincaré period, which is equal to the period of the original system (1), see Eq. (2). A fixed or m-periodic point corresponds to a periodic solution of Eq. (1) with period 2π or $2m\pi$, respectively. Moreover all topological properties of the solution of Eq. (1) can be reduced to that of the discrete dynamical system (5). In this paper we consider following two problems:

I. Assuming that Eq. (1) has a chaotic attractor, design a controller stabilizing a specific periodic orbit, also called a target orbit, embedded in the chaotic attractor.

II. Assuming that Eq. (1) has a stable periodic orbit, destabilize it by a suitable controller.

Note that these two problems are the same if the pole assignment technique is used. Stabilization and destabilization correspond to the stable and unstable pole assignment, respectively. Note also that in the following we shall discuss the stabilization or destabilization method for Eq. (1), but this method can be easily applied to an autonomous ODE by changing the definition of the Poincaré mapping.

3. Stabilizing Unstable Periodic Orbit with a Fixed Point

Let us consider an unstable periodic solution $\boldsymbol{x}^*(t)$ of Eq. (1) with period 2π, which is our target orbit embedded in a chaotic attractor. Suppose that $\boldsymbol{x}^*$ is a fixed point of T, which corresponds to the target orbit $\boldsymbol{x}^*(t)$:

$$\boldsymbol{x}^* = T(\boldsymbol{x}^*, \boldsymbol{\lambda}^*) = \varphi(2\pi, \boldsymbol{x}^*, \boldsymbol{\lambda}^*) \tag{6}$$

where we denote $\boldsymbol{\lambda}^*$ as the nominal value of the parameter. For any integer k, let us consider the perturbations:

$$\boldsymbol{x}(2\pi k) = \boldsymbol{x}_k = \boldsymbol{x}^* + \boldsymbol{\xi}(k), \quad \boldsymbol{\lambda}_k = \boldsymbol{\lambda}^* + \boldsymbol{u}(k). \tag{7}$$

After one iteration of T we have

$$\begin{aligned} \boldsymbol{x}(2\pi(k+1)) &= \boldsymbol{x}_{k+1} = \boldsymbol{x}^* + \boldsymbol{\xi}(k+1) \\ &= T(\boldsymbol{x}^* + \boldsymbol{\xi}(k), \boldsymbol{\lambda}^* + \boldsymbol{u}(k)) \\ &= T(\boldsymbol{x}^*, \boldsymbol{\lambda}^*) + \left.\frac{\partial T}{\partial \boldsymbol{x}}\right|_{\substack{\boldsymbol{x} = \boldsymbol{x}^* \\ \boldsymbol{\lambda} = \boldsymbol{\lambda}^*}} \boldsymbol{\xi}(k) \\ &\quad + \left.\frac{\partial T}{\partial \boldsymbol{\lambda}}\right|_{\substack{\boldsymbol{x} = \boldsymbol{x}^* \\ \boldsymbol{\lambda} = \boldsymbol{\lambda}^*}} \boldsymbol{u}(k) + \cdots \end{aligned} \tag{8}$$

Therefore we obtain the difference equation defined by the derivative of T:

$$\boldsymbol{\xi}(k+1) = \boldsymbol{A}\boldsymbol{\xi}(k) + \boldsymbol{B}\boldsymbol{u}(k) \tag{9}$$

where we put

$$\boldsymbol{A} = \left.\frac{\partial T}{\partial \boldsymbol{x}}\right|_{\substack{\boldsymbol{x} = \boldsymbol{x}^* \\ \boldsymbol{\lambda} = \boldsymbol{\lambda}^*}} \text{ and } \boldsymbol{B} = \left.\frac{\partial T}{\partial \boldsymbol{\lambda}}\right|_{\substack{\boldsymbol{x} = \boldsymbol{x}^* \\ \boldsymbol{\lambda} = \boldsymbol{\lambda}^*}} \tag{10}$$

Note that if $\boldsymbol{u}(k) = 0$, the origin is the unstable fixed point of Eq. (9) provided that x^* is embedded in a chaotic attractor.

Now we construct a state feedback control to stabilize the origin:

$$\boldsymbol{u}(k) = \boldsymbol{C}^\top \boldsymbol{\xi}(k), \tag{11}$$

where $\boldsymbol{C}$ is an $r \times n$ matrix must be designed. Substituting Eq. (11) into Eq. (9) we have

$$\boldsymbol{\xi}(k+1) = [\boldsymbol{A} + \boldsymbol{B}\boldsymbol{C}^\top]\boldsymbol{\xi}(k). \tag{12}$$

By the linear control theory, especially by the pole assignment technique, we can choose an appropriate matrix $\boldsymbol{C}$ to stabilize the origin, equivalently say the fixed point $\boldsymbol{x}^*$, provided that the controllability condition is satisfied [6]:

$$\text{rank}[\boldsymbol{B}|\boldsymbol{A}\boldsymbol{B}|\cdots|\boldsymbol{A}^{n-1}\boldsymbol{B}] = n \tag{13}$$

Hence we obtain the following theorem:

Theorem 1: Let $\boldsymbol{x}^*$ be an unstable fixed point of the mapping T, which corresponds to an unstable periodic solution $\boldsymbol{x}^*(t)$ of Eq. (1) with the period 2π. Assume that the controllability condition (13) is satisfied. Then we can choose a matrix $\boldsymbol{C}$ such that Eq. (9) becomes stable, i.e., the matrix

$$\boldsymbol{A} + \boldsymbol{B}\boldsymbol{C}^\top \tag{14}$$

is stable. Moreover by applying piecewise constant control $\boldsymbol{u}(k) = \boldsymbol{C}^\top\boldsymbol{\xi}(k)$ to the parameter $\boldsymbol{\lambda}$:

$$\begin{aligned} \boldsymbol{\lambda} &= \boldsymbol{\lambda}^* + \boldsymbol{u}(k) = \boldsymbol{\lambda}^* + \boldsymbol{C}^\top\boldsymbol{\xi}(k) \\ &= \boldsymbol{\lambda}^* + \boldsymbol{C}^\top\{\boldsymbol{x}(2\pi k) - \boldsymbol{x}^*\} \\ &\text{with} \quad 2\pi k \leqq t < 2\pi(k+1) \end{aligned} \tag{15}$$

the periodic solution $\boldsymbol{x}^*(t)$ becomes stable.

Remark 1: $\boldsymbol{A}$ and $\boldsymbol{B}$ in Eq. (10) are obtained numerically by solving the following linear ODEs from $t = 0$ to $t = 2\pi$:

$$\begin{aligned} \frac{d}{dt}\boldsymbol{A} &= \left.\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{x}}\right|_{\substack{\boldsymbol{x} = \boldsymbol{x}^* \\ \boldsymbol{\lambda} = \boldsymbol{\lambda}^*}} \boldsymbol{A} \\ \frac{d}{dt}\boldsymbol{B} &= \left.\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{x}}\right|_{\substack{\boldsymbol{x} = \boldsymbol{x}^* \\ \boldsymbol{\lambda} = \boldsymbol{\lambda}^*}} \boldsymbol{B} + \left.\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{\lambda}}\right|_{\substack{\boldsymbol{x} = \boldsymbol{x}^* \\ \boldsymbol{\lambda} = \boldsymbol{\lambda}^*}} \end{aligned} \tag{16}$$

with initial condition

$$\boldsymbol{A}|_{t=0} = I, \quad \boldsymbol{B}|_{t=0} = 0, \tag{17}$$

where I is an identity matrix. Therefore, any analytic or estimating method to obtain $\boldsymbol{A}$ and $\boldsymbol{B}$ is not necessary.

Remark 2 [7]**:** A schematic block diagram of this control is illustrated in Fig. 1.

In this figure sampler operates at every instant $2\pi k$, say the Poincaré sampling instant. At every instant $2\pi k$, the sampled signal $x(2\pi k)$ is clamped during the Poincaré period or until next sampling is achieved.

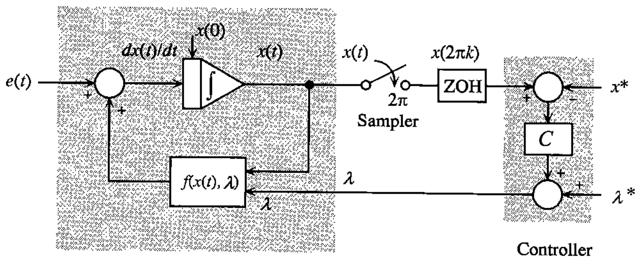

Fig. 1 Composite dynamical system.

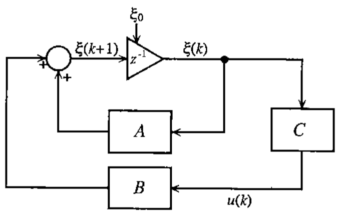

Fig. 2 State feedback control described by Eqs. (9) and (11).

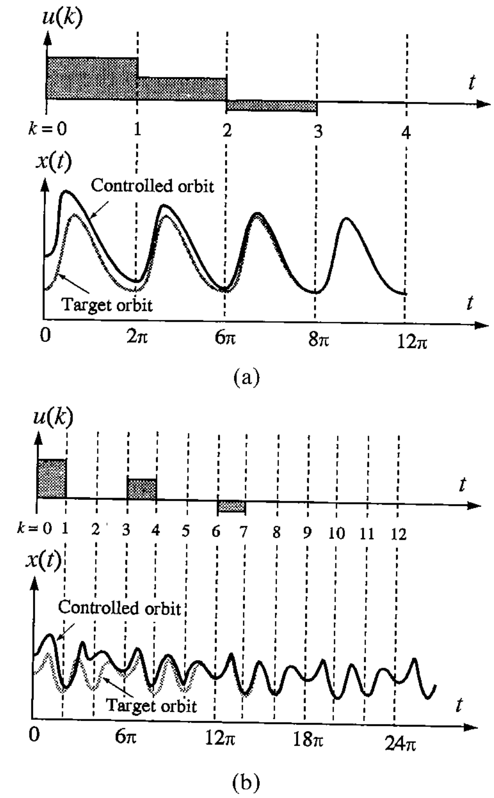

Fig. 3 Response of the control signal and state $x(t)$. (a) Fixed point, (b) 3-periodic point.

Therefore the parameter $\boldsymbol{\lambda}$ is changed discontinuously at every instant $2\pi k$. Such a sampled data system with zero-order hold (clamper; ZOH) is the most commonly known sampled data system. Note that, however, for practical digital control system the sampled interval is chosen as much shorter than our Poincaré period.

Remark 3: Eq. (9) is used only for designing the control matrix $\boldsymbol{C}$. Therefore it does not explicitly appear in the diagram shown in Fig. 1. We use the linear difference equation to stabilize the origin at the designing process, see Fig. 2. If $\boldsymbol{C}$ is calculated once for all, the information about the difference equation can be removed. Design steps of the matrix $\boldsymbol{C}$ for pole assignment are omitted here, but it is easily found in linear control textbooks, e.g., see Ref. [8].

Remark 4: The control signal (15) may be started to apply to Eq. (1) when an orbit wandering in the chaotic attractor passes through in the neighborhood of $\boldsymbol{x}^*$. A detecting element, called watcher, measures

$$||\boldsymbol{x}(2\pi k) - \boldsymbol{x}^*|| < \epsilon \tag{18}$$

at every Poincaré sampling instant, and switches the control signal.

The controlled system is then totally described by

$$\begin{aligned} &\frac{d\boldsymbol{x}(t)}{dt} = \boldsymbol{f}\left(t, \boldsymbol{x}(t), \boldsymbol{\lambda}^* + \boldsymbol{C}^\top \boldsymbol{\xi}(k)\right), \\ &\qquad \text{for} \quad 2\pi k \leqq t < 2\pi(k+1) \\ &\boldsymbol{\xi}(k) = [\boldsymbol{A} + \boldsymbol{B}\boldsymbol{C}^\top]\boldsymbol{\xi}(k-1), \\ &\qquad \text{at} \quad t = 2\pi k \end{aligned} \tag{19}$$

with $\boldsymbol{\xi}(k) = \boldsymbol{x}(2\pi k) - \boldsymbol{x}^*$ for every integer k. This is a mixed continuous and discrete dynamical system, which we call a composite dynamical system (CDS). The most significant property of Eq. (19) is that the stability of the solutions $\boldsymbol{x}^*(t)$ and $\boldsymbol{\xi}(k) = \boldsymbol{0}$ is determined by the discrete part:

$$\boldsymbol{\xi}(k) = [\boldsymbol{A} + \boldsymbol{B}\boldsymbol{C}^\top]\boldsymbol{\xi}(k-1) \tag{20}$$

Hence if $\boldsymbol{A} + \boldsymbol{B}\boldsymbol{C}^\top$ is designed to be stable, then $(\boldsymbol{\xi}(k), \boldsymbol{u}(k))$ tends to $(0, 0)$ as $k \to \infty$. This means at the steady state the parameter $\boldsymbol{\lambda}$ seems to be invariant as $\boldsymbol{\lambda}^*$ so that $\boldsymbol{x}^*$ is exactly the same trajectory as the uncontrolled original system, see Fig. 3 (a). Note that the solution $\boldsymbol{x}(t)$ is unstable if we consider only the first equation of (19), but it becomes stable in the CDS.

4. Stabilizing Unstable m-Periodic Orbit

Similar result can be easily obtained for m-periodic point of T. Let the following m points:

$$\left\{ \begin{array}{l} \boldsymbol{x}^*(2\pi) = \boldsymbol{x}_1^* = T(\boldsymbol{x}_m^*, \boldsymbol{\lambda}^*), \\ \boldsymbol{x}^*(2\pi k) = \boldsymbol{x}_k^* = T(\boldsymbol{x}_{k-1}^*, \boldsymbol{\lambda}^*), \\ \qquad \text{for} \quad k = 2, 3, \cdots, m. \end{array} \right. \tag{21}$$

be m-periodic points of T. This means that

$$\boldsymbol{x}_k^* = T^m(\boldsymbol{x}_k^*, \boldsymbol{\lambda}^*), \quad \text{for} \quad k = 1, 2, \cdots, m. \tag{22}$$

holds, i.e., $\boldsymbol{x}_k^*$ is the fixed point of T^m. Hence choosing the Poincaré period as $2m\pi$, we can construct the control matrix $\boldsymbol{C}$ at every $2\pi mk$ instant, see Fig. 3 (b). For more detailed information, see Ref. [6].

5. Destabilizing Stable Fixed or m-periodic Point

In this section we discuss to design a controller destabilizing a fixed point or m-periodic of the mapping T. We have the following destabilizing theorem.

Theorem 2: Let $\boldsymbol{x}^*$ be a stable fixed point of the mapping T, which corresponds to a stable periodic solution $\boldsymbol{x}(t)$ of Eq. (1) with the period 2π. Assume that the controllability condition (13) is satisfied. Then we can choose a matrix $\boldsymbol{C}$ such that Eq. (9) becomes unstable, i.e., the matrix

$$\boldsymbol{A} + \boldsymbol{B}\boldsymbol{C}^\top \tag{23}$$

is unstable. Moreover by applying piecewise constant control $\boldsymbol{u}(k) = \boldsymbol{C}^\top \boldsymbol{\xi}(k)$ to the parameter $\boldsymbol{\lambda}$:

$$\begin{aligned} \boldsymbol{\lambda} &= \boldsymbol{\lambda}^* + \boldsymbol{u}(k) = \boldsymbol{\lambda}^* + \boldsymbol{C}^\top \boldsymbol{\xi}(k) \\ &= \boldsymbol{\lambda}^* + \boldsymbol{C}^\top \{\boldsymbol{x}(2\pi k) - \boldsymbol{x}^*\}, \\ &\quad \text{for} \quad 2\pi k \leqq t < 2\pi(k+1) \end{aligned} \tag{24}$$

the periodic solution $\boldsymbol{x}^*(t)$ becomes unstable.

Remark 5: After the destabilization the orbit of CDS (19) may be chaotic, periodic, or convergent to another stable attractor.

6. Target Generating and Noise Effect

To calculate a target (fixed or periodic point), the Newton's method using the Jacobian matrix $\boldsymbol{A}$ is available. Any precision of the target location can be obtained unless that the Jacobian matrix $\boldsymbol{A}$ is singular by this method. When a deterministic differential equation is given, the local properties of the orbit is completely described by $\boldsymbol{A}$, which is the solution of the first equation of Eqs. (16). Therefore, if the target can be calculated by $\boldsymbol{A}$ and the condition (13) is held, control can be succeed by suitable choice of ϵ.

The width of ϵ giving the control available region called basin of attraction depends on stability of the target, control parameters, and assigned poles [6], especially, the basin tends to reduce as the period of the target becomes higher. This disadvantage causes that the transient chaotic response is too long. If the basin of attraction can be wide, not only suppressing for transient responses but also the robustness of the control against disturbances or noise is earned. We must investigate to enlarge the basin in future for physical implementations.

7. Illustrated Examples

We choose Duffing's equation:

$$\begin{aligned} \frac{dx}{dt} &= y \\ \frac{dy}{dt} &= -\kappa y - x^3 + B_0 + B\cos t \end{aligned} \tag{25}$$

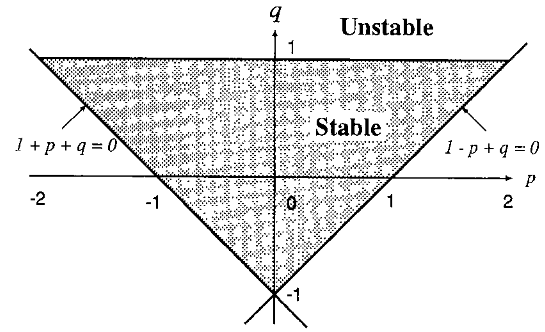

Fig. 4 Stability in p-q plane for Eq. (26).

to demonstrate our method of control [7]. For simplicity in this paper we assume that we know the system, i.e., we can obtain all information for constructing the control matrix $\boldsymbol{C}$. Hence unstable fixed or periodic point embedded in some chaotic attractor is calculated in advance by using Newton's method. Matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ are also calculated from Eq. (16).

Now we choose B_0 as the control parameter and calculate control vector $\boldsymbol{C}(2 \times 1)$. Any other parameter can be chosen if the condition Eq. (13) is held. To determine the control vector $\boldsymbol{C}$ we consider the characteristic equation from Eq. (14):

$$|\boldsymbol{A} + \boldsymbol{B}\boldsymbol{C}^\top - \mu I| = \mu^2 - p\mu + q = 0. \tag{26}$$

Thus the modulus of the root of Eq. (26) is less than unity if p and q are placed in the triangle such that:

$$\begin{aligned} &q < 1 \\ &1 + p + q > 0 \\ &1 - p + q > 0. \end{aligned} \tag{27}$$

Figure 4 shows the stable region in p-q plane.

7.1 Stabilization

Example 1: Let us consider the case where $\kappa = 0.02$, $B_0 = 2.0$, and $B = 2.2$ in Eqs. (25). The equations have a chaotic attractor shown in Fig. 5 (a). In the chaotic attractor we see unstable fixed or periodic points. Some of them are listed in Table 1. We calculate the control vector $\boldsymbol{C}$ so as $p = q = 0$ which gives a dead beat control. In this case $\boldsymbol{A} + \boldsymbol{B}\boldsymbol{C}^\top$ becomes a nilpotent matrix. Hence for any initial condition $\boldsymbol{\xi}(0)$, the state $\boldsymbol{\xi}(k)$ falls into 0 at most twice iteration of $\boldsymbol{A} + \boldsymbol{B}\boldsymbol{C}^\top$.

Figures 5 (b)–(f) show the stabilized periodic solutions by the controlling (15). For the fixed point of Fig. 5 (b), this is given by:

$$\begin{aligned} B_0 = 2.0 &+ 2.3071(x(2\pi k) - 2.3891) \\ &+ 0.3784(y(2\pi k) - 0.0256). \end{aligned} \tag{28}$$

Although the control parameter B_0 changes discontinuously, if B is chosen as the control parameter and its controllability is held, the external force $B_0 + (B + u(2\pi)k)\cos t$ can be smooth.

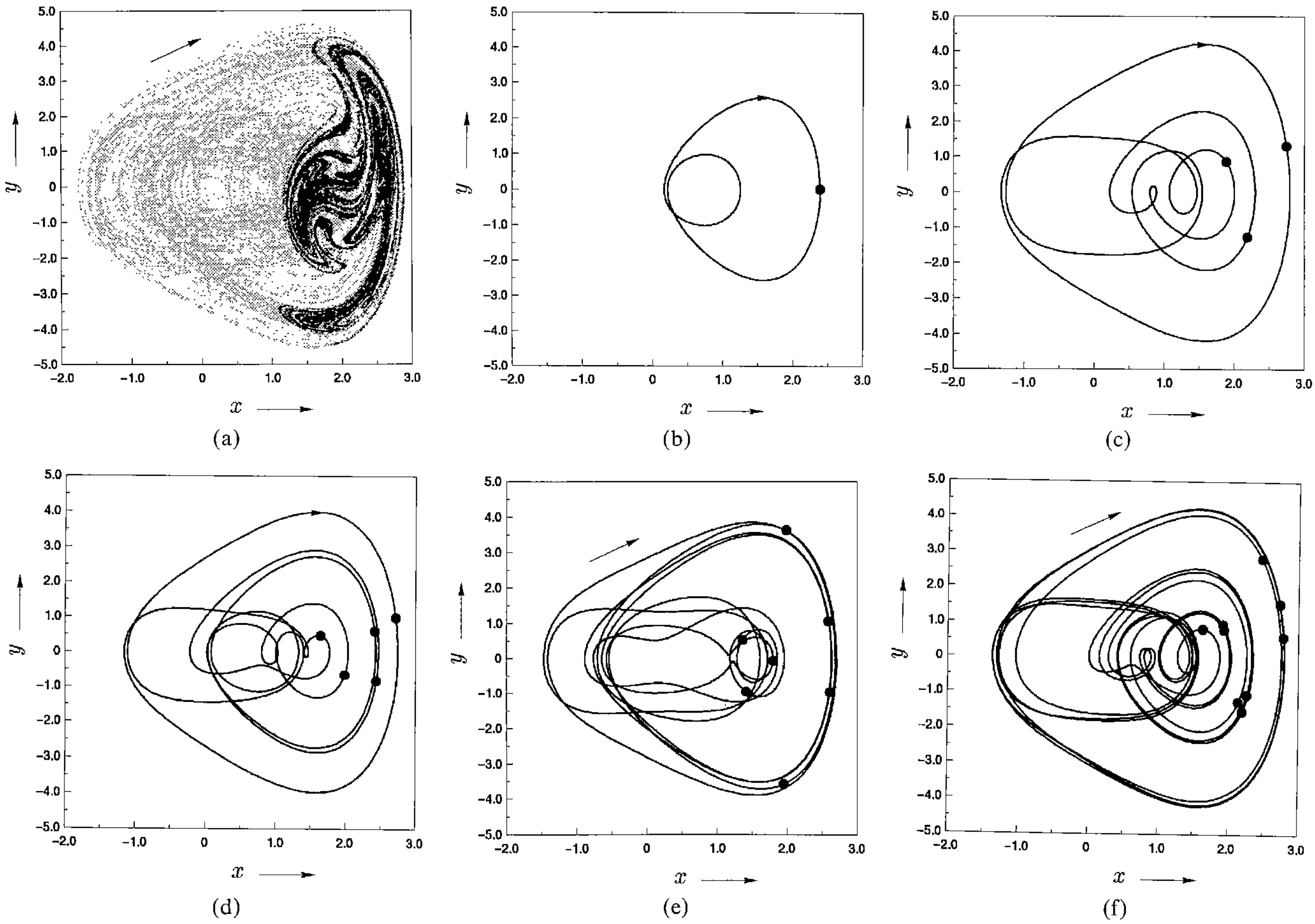

Fig. 5 Chaotic attractor and stabilized orbits embedded in the attractor. $\kappa = 0.02$, $B_0 = 2.0$, $B = 2.2$. (a) Chaotic attractor (orbit and Poincaré mapping), (b) Fixed point, $\epsilon = 0.1$ (c) Period-3, $\epsilon = 0.1$, (d) Period-5, $\epsilon = 0.1$, (e) Period-7, $\epsilon = 0.05$, (f) Period-9, $\epsilon = 0.01$

Table 1 Fixed or periodic points and the values of control vectors.

figure	period	fixed/periodic point	eigenvalues	control vector C
Fig. 5 (b)	1	$(2.3891, 0.0256)$	$(-0.1776, -4.9633)$	$[2.3071 \ \ 0.3784]$
Fig. 5 (c)	3	$(2.1868, -1.2681)$	$(-0.2681, -2.5579)$	$[2.4317 \ \ 0.0140]$
Fig. 5 (d)	5	$(2.4427, -0.8255)$	$(-0.01366, -39.0295)$	$[2.3561 \ \ 0.0673]$
Fig. 5 (e)	7	$(1.9857, 3.6287)$	$(-0.02167, -19.1425)$	$[4.1572 \ \ 4.1529]$
Fig. 5 (f)	9	$(1.9391, 0.7922)$	$(0.005869, 54.9848)$	$[0.9811 \ \ -0.8679]$

7.2 Destabilization

Example 2: Now we consider the case where $\kappa = 0.1$, $B_0 = 0.0$, and $B = 0.3$. In this case we have two stable fixed points ${}^1S$ and ${}^2S$ as shown in Fig. 6 (a). Let us try the destabilization of the fixed point ${}^2S$ in Fig. 6 (a) by placing the poles of Eq. (26) out of the triangle Eq. (27) so that solution enters in the basin of the stable fixed point ${}^1S$ and finally tends to ${}^1S$. The transient process is shown in Fig. 6 (b). This example suggests that the destabilization method may be efficiently applied to escape from a local minimum in various optimization problems.

Example 3: As the final example we consider the case where Eq. (25) have only one stable fixed point. In this case we may observe a chaotic attractor by choosing C appropriately. Two examples are shown in Figs. 7 (a) and (b). Both attractors have positive Lyapunov exponents, see Figs. 7.

8. Concluding Remarks

We propose the stabilization and destabilization method of periodic orbits for continuous dynamical system described ODE. In our stabilization method all trajectories are remain to be continuous and converge to a specific unstable orbits. Dead beat control design is one of the conventional method for this purpose. Many

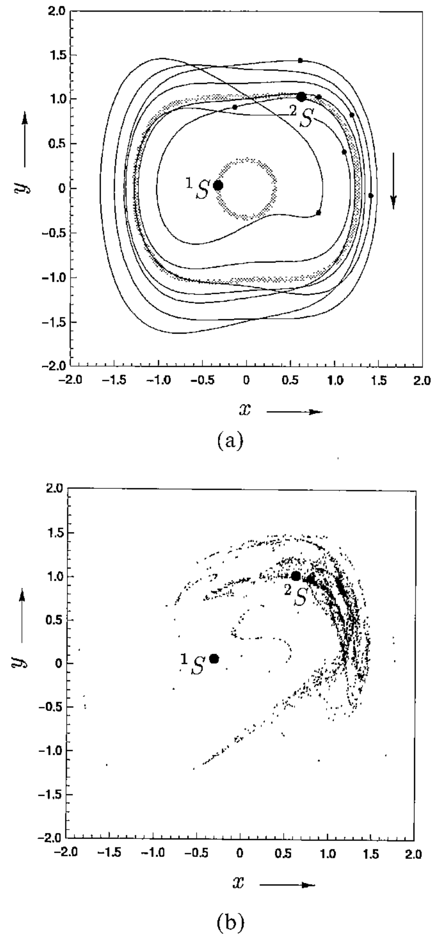

Fig. 6 (a) Created 7-periodic orbit by destabilizing the fixed point ^2S. Poles are -4.0 and 4.0, $\epsilon = 0.3$. (b) Created chaotic transient eventually falling into the basin of ^1S. Poles are 6.6 and 0.0, $\epsilon = 0.3$

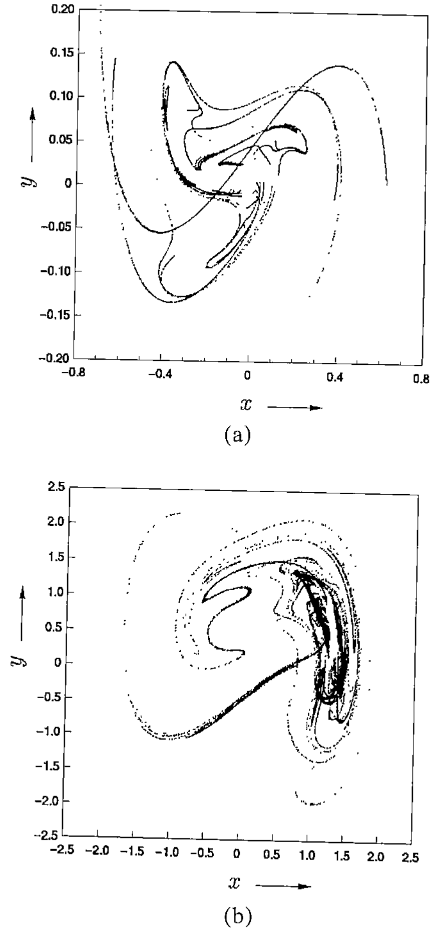

Fig. 7 Created chaotic attractor. (a) $B = 0.1$, Poles are -9.0 and 0.0, $\epsilon = 0.1$. (b) $B = 0.5$. Poles are -12.0 and 6.0, $\epsilon = 0.3$. The maximum Lyapunov exponents of these examples are 0.3471, 0.1303, respectively.

techniques known in the linear control theory can be applied to design the control matrix $\boldsymbol{C}$. For examples an output feedback method with observer, and optimal control method are directly applied to our problems. For controlling chaotic signal generated from unknown system we must construct target (the location of fixed point etc.) and matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ only by using the chaotic signal. For the practical application this type of question is an interesting problem left to the future.

References

[1] Ott, E., Grebogi, C. and Yorke, J., "Controlling Chaos," *Phys. Rev. Lett.*, vol.64, no.11, pp.1196–1199, 1989.

[2] Romeiras, F., Grebogi, C., Ott, E. and Dayawansa, W., "Controlling Chaotic Dynamical Systems," *Physica*, D, vol.58, pp.165–192, 1992.

[3] Ogorzalek, M., "Taming Chaos: Part II—Control," *IEEE Trans.*, vol.CAS-40, no.10, pp.700–706, 1993.

[4] Chen, G. and Dong, X., "On Feedback Control of Chaotic Continuous-Time Systems," *IEEE Trans.*, vol.CAS-40, no.9, pp.591–601, 1993.

[5] Pyragas, K., "Continuous Control of Chaos by Self-Controlling Feedback," *Phys. Lett.*, A, vol.170, pp.421–428, 1992.

[6] Kawakami, H. and Ueta, T., "Controlling Theory as Applied to Controlling Chaotic Dynamical Systems," *IECE Tech. Rep.*, NLP94-63, 1994.

[7] Ueta, T. and Kawakami, H., "Stabilization Method of Controlling Chaotic Systems," *IECE Tech. Rep.*, NLP94–62, 1994.

[8] Ogata, K., *Modern Control Engineering*, 2nd. edition, Prentice-Hall International, 1990.

Tetsushi Ueta was born in Kochi, Japan, on March 1, 1967. He received the B.Eng. in Electronic engineering, and M.Eng. in Electrical Engineering from the University of Tokushima, Tokushima, Japan, in 1990, and 1992, respectively. Since 1992, he was been Research Associate of Information Science and Intelligent Systems, the University of Tokushima. His interest is bifurcation problems of dynamics.

Hiroshi Kawakami was born in Tokushima, Japan, on December 6, 1941. He received the B.Eng. degree from the University of Tokushima, Tokushima, Japan, in 1964, the M.Eng. and Dr.Eng. degrees from Kyoto University, Kyoto, Japan, in 1966 and 1974, respectively, all in electrical engineering. Presently, he is Professor of Electrical and Electronic Engineering, the University of Tokushima, Tokushima, Japan. His interest is qualitative properties of nonlinear circuits.