PAPER

# Performance of Chaos and Burst Noises Injected to the Hopfield NN for Quadratic Assignment Problems

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**SUMMARY** In this paper, performance of chaos and burst noises injected to the Hopfield Neural Network for quadratic assignment problems is investigated. For the evaluation of the noises, two methods to appreciate finding a lot of nearly optimal solutions are proposed. By computer simulations, it is confirmed that the burst noise generated by the Gilbert model with a laminar part and a burst part achieved the good performance as the intermittency chaos noise near the three-periodic window.
***key words:*** *chaos, intermittency, burst noise, neural network, combinatorial optimization problems, QAP*

## 1. Introduction

Solving combinatorial optimization problem is one of the important applications of the neural network (abbr. NN). If we choose connection weights between neurons of the Hopfield NN appropriately according to given problems, we can obtain a good solution by the energy minimization principle [1]. However, the solutions are often trapped into a local minimum and do not reach the global minimum (namely optimal solution). In order to avoid this critical problem, several people proposed the method adding some kinds of noise for solving traveling salesman problems (abbr. TSP) with the Hopfield NN. Hayakawa and Sawada pointed out that intermittency chaos near the three-periodic window of the logistic map gains the best performance [2]. They concluded that the good result might be obtained by some properties of the chaos noise; short time correlations of the time-sequence. Hasegawa et al. investigated solving abilities of the Hopfield NN with various surrogate noises, and they concluded that the effects of the chaotic sequence for solving optimization problems can be replaced by stochastic noise with the similar autocorrelation [3]. However, their researches treated relatively easy problem (TSP) only and they evaluated the performance of the noises by whether to find the optimal solution or not. We consider that the performance of the noises should be evaluated by finding a lot of nearly optimal solutions within a certain period, especially for difficult problems. Hence, we do not consider that the reason of good performance of chaos noise has been completely disclosed. Especially, we emphasize that it is important to probe into the reason why the intermittency chaos is better than fully developed chaos.

In this paper, the intermittency chaos noise near the three-periodic window is modeled by the Gilbert model [4] with a laminar part and a burst part. Both of intermittency chaos noise and the burst noise generated by the Gilbert model are injected to the Hopfield NN for quadratic assignment problems (abbr. QAP) said to be much more difficult to solve than TSP [5], [6]. We examine frequency distribution of the obtained solutions to compare the performance of the noises in detail. Furthermore, we propose two methods *Depth_1* and *Depth_2* to appreciate finding a lot of nearly optimal solutions for the evaluation of the noises. By computer simulations we confirm that the burst noise generated by the Gilbert model is also effective to solve QAP and we can say that the irregular switching of the laminar parts and the burst parts is one reason of the good performance of the Hopfield NN with chaos noise.

## 2. Solving QAP with the Hopfield NN

Various methods are proposed for solving QAP which is one of the NP-hard combinatorial optimization problems. QAP is expressed as follow: given two matrices, distance matrix $\mathbf{C}$ and flow matrix $\mathbf{D}$, and find the permutation $\mathbf{p}$ which corresponds to the minimum value of the objective function $f(\mathbf{p})$ in Eq. (1).

$$f(\mathbf{p}) = \sum_{i=1}^{N} \sum_{j=1}^{N} C_{ij} D_{p(i)p(j)}, \tag{1}$$

where $C_{ij}$ and $D_{ij}$ are the $(i, j)$-th elements of $\mathbf{C}$ and $\mathbf{D}$, respectively, $p(i)$ is the $i$-th element of the vector $\mathbf{p}$, and $N$ is the size of the problem. There are many real applications which are formulated by Eq. (1). One example of QAP is to find an arrangement of factories to make a cost the minimum. The cost is given by the distance between the cities and the flow of the products between the factories (Fig. 1).

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Other examples are the placement of logical modules in an IC chip, the distribution of medical services in a large hospital, and so on.

Because the QAP is very difficult, it is almost impossible to solve the optimum solutions in larger problems. The largest problem whose optimal solution can be obtained may be only 36 in recent study [7]. Further, computation time is very long to obtain the exact optimum solutions. Therefore, it is usual to develop heuristic methods which search nearly optimal solutions in reasonable time.

For solving an $N$-element QAP by the Hopfield NN, $N \times N$ neurons are required and the following energy function is defined:

$$E = \sum_{i,m=1}^{N} \sum_{j,n=1}^{N} w_{im;jn} x_{jn} + \sum_{i,m=1}^{N} \theta_{im} x_{im}. \tag{2}$$

The neurons are coupled each other with the synaptic connection weight. Suppose that the weight between $(i, m)$-th neuron and $(j, n)$-th neuron and the threshold of the $(i, m)$-th neuron are described by:

$$\begin{aligned} w_{im;jn} &= -2\left\{A(1-\delta_{mn})\delta_{ij} \right. \\ &\qquad \left. + B\delta_{mn}(1-\delta_{ij}) + \frac{C_{ij}D_{mn}}{q}\right\} \\ \theta_{im} &= A + B \end{aligned} \tag{3}$$

where $A$ and $B$ are positive constants, $q$ is a normalization parameter to correspond given problems, and $\delta_{ij}$ is the Kronecker's delta. The states of $N \times N$ neurons are asynchronously updated due to the following difference equation:

$$x_{im}(t+1) = g\left(\sum_{j,n=1}^{N} w_{im;jn} x_{jn}(t) - \theta_{im} + \beta z_{im}(t)\right) \tag{4}$$

where $g$ is a sigmoidal function defined as follows:

$$g(x) = \frac{1}{1 + \exp\left(-\dfrac{x}{\varepsilon}\right)} \tag{5}$$

$z_{im}$ is an additional noise, and $\beta$ limits the amplitude of the noise.

Also, we use the method suggested by Sato et al. (1.1 in [8]) to decide firing of neurons.

## 3. Chaos and Burst Noises

### 3.1 Chaos Noise

In this subsection, we describe chaos noise injected to the Hopfield NN. The logistic map is used to generate the chaos noise:

$$\hat{z}_{im}(t+1) = \alpha \hat{z}_{im}(t)(1 - \hat{z}_{im}(t)). \tag{6}$$

Varying the parameter $\alpha$, Eq. (6) behaves chaotically via a period-doubling cascade. Further, it is well known that the map produces intermittent bursts just before periodic-windows appear. Figure 2 shows an example of the intermittency chaos near the three-periodic window obtained from Eq. (6) for $\alpha = 3.8274$. As we can see from the figure, the chaotic time series could be divided into two phases; laminar parts of periodic behavior with period three and burst parts of spread points over the invariant interval. As increasing $\alpha$, the ratio of the laminar parts becomes larger and finally the three-periodic window appears. For the comparison, we also carry out computer simulations for the case of fully developed chaos in Fig. 3 which is obtained from Eq. (6) for $\alpha = 4.0000$. This noise is much more similar to the random noise.

When we inject the chaos noises to the Hopfield NN, we normalize $\hat{z}_{im}$ by Eq. (7).

$$z_{im}(t) = \frac{\hat{z}_{im}(t) - \bar{z}}{\sigma_z} \tag{7}$$

where $\bar{z}$ is the average of $\hat{z}(t)$, and $\sigma_z$ is the standard deviation of $\hat{z}(t)$.

### 3.2 Burst Noise

In this subsection, we model the intermittency chaos by using the two-state Gilbert model shown in Fig. 4. The Gilbert model is sometimes used for characterizing error-generating mechanisms in digital communication channels.

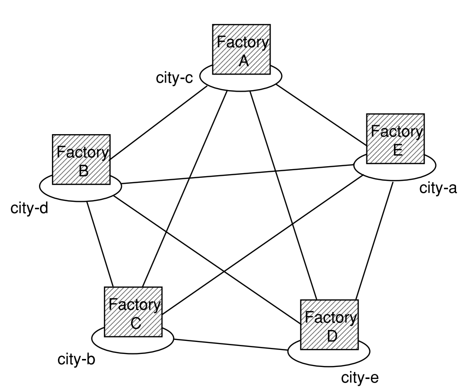


**Fig. 1** An example of QAP.

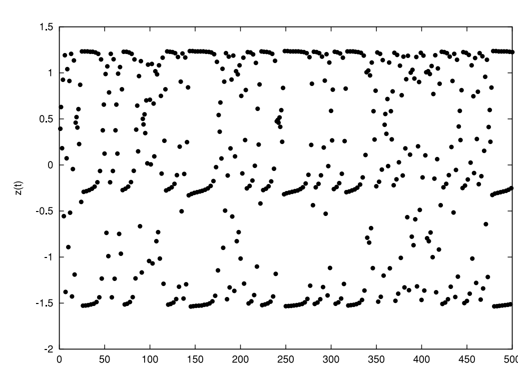


**Fig. 2** Intermittency chaos noise near three-periodic window. $\alpha$=3.8274.

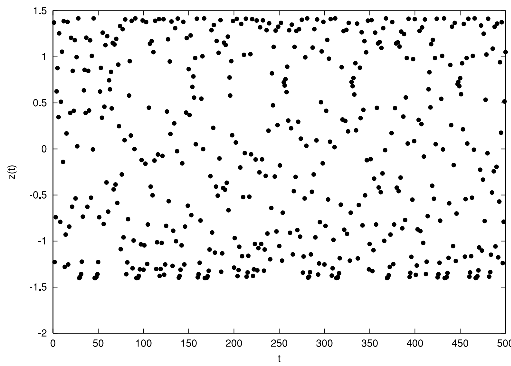

**Fig. 3** Fully developed chaos noise. $\alpha$=4.0000.

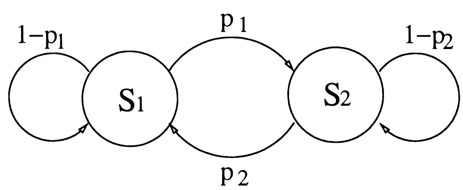


**Fig. 4** Two-state Gilbert model.

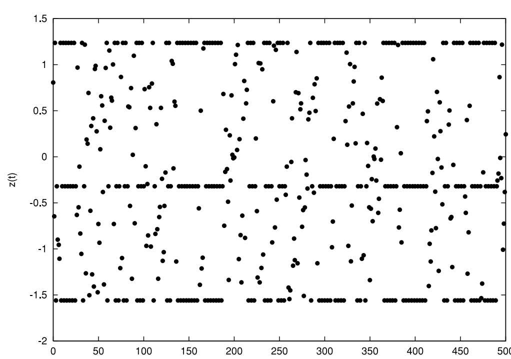

**Fig. 5** Burst noise generated by the Gilbert model with $p_1$ = 0.247 and $p_2$ = 0.238.

At first, we distinguish the laminar part and the burst part of the intermittency chaos. Because we treat only the intermittency chaos near the three-periodic window, we regard three successive sequences starting from a point whose value is 1.194 or more as one-period of the laminar part. Other points are regarded as the burst part.

One state $S_1$ corresponds to the burst part and generates uniformly distributed noise over the interval {−1.56–1.236}, while the other state $S_2$ corresponds to the laminar part and generates three-periodic sequence {1.236, −1.560, −0.320} imitating the three-periodic window of the logistic map. The transition probabilities $p_1$ and $p_2$ are calculated by the average lengths of the laminar parts and the burst parts in the intermittency chaos in Fig. 2 according to

$$\text{Average length of the burst parts} = \frac{1}{p_1} \quad (8)$$

$$\text{Average length of the laminar parts} = \frac{1}{p_2} \quad (9)$$

Figure 5 shows an example of the burst noise generated by the Gilbert model.

## 4. Simulated Results

In this section, the simulated results of the Hopfield NN for QAP with the three different noises, namely the intermittency chaos noise, the fully developed chaos noise, and the burst noise generated by the Gilbert model are shown.

The first target problem is named as "Nug12" chosen from the site QAPLIB [7]. The number of the elements is 12 and the optimal solution is known as 578. The parameters of the Hopfield NN are fixed as $A = 0.9$, $B = 0.9$, $q = 140$, and $\varepsilon = 0.02$ and the amplitude of the injected noise is fixed as $\beta = 0.6$. The iteration number $N_{iteration}$ of the network is fixed as 10000.

### 4.1 Solving Ability

Since we know the optimal solution in advance, we define the **Success** such that the NN finds the optimal solution at least once during the defined iteration number $N_{iteration}$. We repeat this trial 100 times and count the number of the **Success** for the solving ability $SA$ defined as

$$SA = \frac{\text{Number of Success}}{\text{Number of Trials}} \times 100\ [\%]. \quad (10)$$

We also record the minimum cost found during each trial and use the **Average** of the minima for the evaluation. The $Error$ is also defined as

$$Error = \frac{\text{Average} - \text{Optimal Solution}}{\text{Optimal Solution}} \times 100\ [\%]. \quad (11)$$

The results are summarized in Table 1. The results show that the intermittency chaos noise and the burst noise have much better performance than the fully developed chaos noise. Furthermore, it is interesting to note that the burst noise generated by the Gilbert model has the similar Average to the intermittency chaos noise.

### 4.2 Frequency Distribution

In order to make clear the reason of the good performance of the intermittency chaos noise and the burst noise, we examine frequency distribution of the obtained solutions.

At first, we explain how to accept the solutions. The Hopfield NN with the noises can find various solutions. However, the state of the Hopfield NN sometimes stays around one solution or a group of several solutions. We consider that such a behavior is not useful to find the optimal or nearly optimal solutions. So, we take the only-different-solutions method. Namely, we take into account only the solutions whose firing patterns have not found ever in each trial. In other words, we accept only the first time if the completely same firing patterns appear more than once in each trial.

The results of the frequency distribution are shown in

**Table 1** Solving abilities for 12-element QAP.

| | Intermittency chaos | | | Burst | | | Fully developed chaos | | |
|---|---|---|---|---|---|---|---|---|---|
| $N_{iteration}$ | $SA$[%] | Average | $Error$[%] | $SA$[%] | Average | $Error$[%] | $SA$[%] | Average | $Error$[%] |
| 1000 | 1 | 612.66 | 5.997 | 1 | 615.52 | 6.491 | 0 | 629.46 | 8.903 |
| 2000 | 1 | 607.64 | 5.128 | 1 | 607.96 | 5.183 | 0 | 628.08 | 8.664 |
| 3000 | 3 | 604.04 | 4.505 | 1 | 604.36 | 4.561 | 0 | 627.98 | 8.647 |
| 4000 | 3 | 602.36 | 4.214 | 1 | 602.34 | 4.211 | 0 | 627.98 | 8.647 |
| 5000 | 5 | 601.02 | 3.983 | 1 | 601.42 | 4.052 | 0 | 627.98 | 8.647 |
| 6000 | 7 | 600.24 | 3.848 | 1 | 600.70 | 3.927 | 0 | 627.98 | 8.647 |
| 7000 | 7 | 599.32 | 3.689 | 2 | 599.76 | 3.765 | 0 | 627.98 | 8.647 |
| 8000 | 7 | 599.08 | 3.627 | 2 | 599.42 | 3.706 | 0 | 627.98 | 8.647 |
| 9000 | 7 | 598.42 | 3.533 | 2 | 598.96 | 3.626 | 0 | 627.98 | 8.647 |
| 10000 | 7 | 598.14 | 3.484 | 2 | 598.74 | 3.588 | 0 | 627.98 | 8.647 |

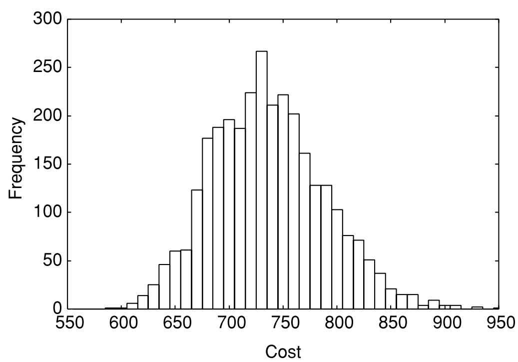


**Fig. 6** Frequency distribution of solutions with intermittency chaos noise for Nug12.

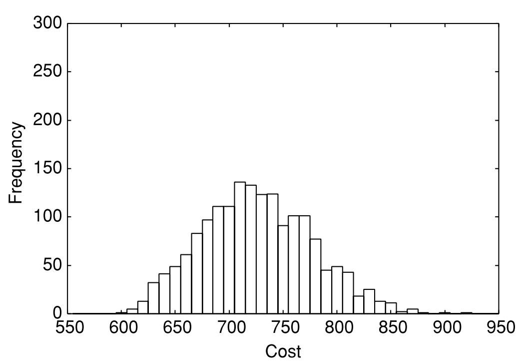


**Fig. 7** Frequency distribution of solutions with burst noise for Nug12.

Figs. 6, 7 and 8. The frequency means the number of the accepted solutions with the corresponding costs found during 10000 iterations. We can see that a lot of solutions are found for the cases of the intermittency chaos noise and the burst noise. On the other hand, only a very small number of the solutions are found for the fully developed chaos. (Note that the scale of the vertical axis of Fig. 8 is 1/10 of those of Figs. 6 and 7.) Although the result in Table 1 would reflect the difference of the numbers of the obtained solutions, we can see that the methods using only the best solution in each trial are not sufficient to evaluate the performances of the noises.

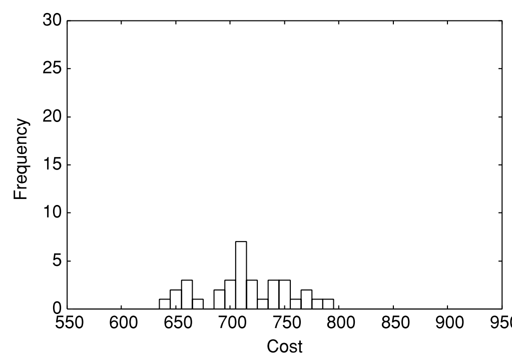


**Fig. 8** Frequency distribution of solutions with fully developed chaos noise for Nug12.

## 4.3 Depth

Until now, we have evaluated the performance of the Hopfield NN with noises by using only the best solution found in each trial; time to find the optimal solution, frequency of finding the optimal solution during a given time, average of the minimum cost in each trial, and so on. However, as shown in the previous subsection, such methods are not sufficient to reflect the number of the obtained solutions. Further, when the network can not find the optimal solution, the methods using only the optimal solution do not give a correct evaluation of the network. Namely, for very difficult problems, we consider that it is more important to find a lot of nearly optimal solutions than to happen to find the optimal solution. Therefore, we propose two evaluation methods to appreciate finding a lot of nearly optimal solutions.

### 4.3.1 *Depth_1*

The first evaluation method *Depth_1* is defined as

$$Depth\_1 = \sum_{k=0}^{n} \{f(\mathbf{p}_k) - D_\infty\}^2 \tag{12}$$

where $D_\infty$ is a constant which is large enough to include the costs of all solutions, $n$ is the number of the accepted solutions and the cost $f(\mathbf{p}_k)$ is calculated by Eq. (1) using the permutation $\mathbf{p}_k$ corresponding to the $k$-th accepted solution.

**Table 2** *Depth_1* for Nug12.

| $D_\infty$ | *Depth_1* | | |
|---|---|---|---|
| | Intermittency chaos | Burst | Fully developed chaos |
| 1000 | 1.07e+08 | 0.80e+08 | 7.36e+06 |

**Table 3** *Depth_2* for Nug12.

| $D_{th}$ | *Depth_2* | | |
|---|---|---|---|
| | Intermittency chaos | Burst | Fully developed chaos |
| 858.8 | 2.58e+07 | 1.99e+07 | 1.87e+06 |
| 812.0 | 1.21e+07 | 0.95e+07 | 9.11e+05 |
| 765.2 | 3.38e+06 | 2.92e+06 | 2.97e+05 |

The calculated result of *Depth_1* is shown in Table 2. The performances of the intermittency chaos noise and the burst noise generated by the Gilbert model are much higher than the fully developed chaos noise. We consider that this result is more appropriate than the evaluation using only the optimal solution to indicate the fact that the fully developed chaos noise is behind the others. Further, we can see that there is still some difference between the intermittency chaos noise and the burst noise.

### 4.3.2 *Depth_2*

The second evaluation method *Depth_2* is defined as

$$Depth\_2 = \sum_{k\in\mathbf{k}_g}^{n} \{f(\mathbf{p}_k) - D_{th}\}^2 - \sum_{k\notin\mathbf{k}_g}^{n} \{f(\mathbf{p}_k) - D_{th}\}^2 \quad \text{where} \quad \mathbf{k}_g = \{k \mid f(\mathbf{p}_k) \le D_{th}\}. \tag{13}$$

This evaluation has an advantage such that we can set the threshold $D_{th}$ according to the requirement. We consider that finding a lot of bad solutions makes the performance of the network worse. However, the value of *Depth_1* increases even if the obtained solution is very bad. Hence, in this evaluation, we not only set up a threshold but give a penalty according to the cost. Namely, if the network finds a solution with the cost more than a given threshold value, the value of *Depth_2* is reduced.

The calculated result of *Depth_2* is shown in Table 3. The tendency of the result is similar to that of *Depth_1*.

## 4.4 30-Element Problem

We carried out computer simulation for a larger problem with 30 elements named "Nug30." The global minimum of this target problem is known as 6124. The parameters of the Hopfield NN are fixed as $A = 0.9$, $B = 0.9$, $q = 640$, and $\varepsilon = 0.02$ and the amplitude of the injected noise is fixed as $\beta = 0.6$. The iteration number $N_{iteration}$ is also fixed as 10000.

The results of the frequency distribution are shown in Figs. 9, 10 and 11. Further, the calculated results of *Depth_1* and *Depth_2* are shown in Tables 4 and 5, respectively.

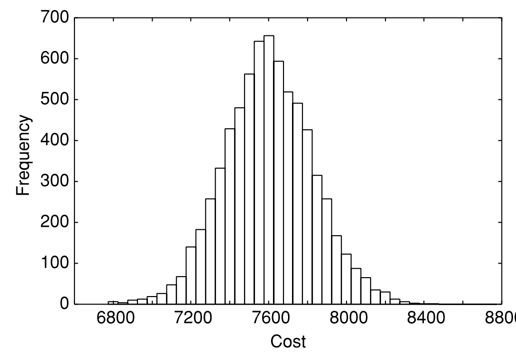


**Fig. 9** Frequency distribution of solutions with intermittency chaos noise for Nug30.

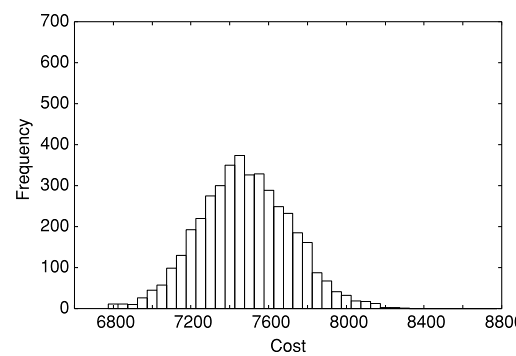


**Fig. 10** Frequency distribution of solutions with burst noise for Nug30.

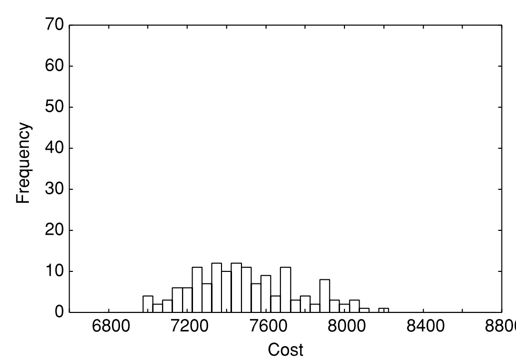


**Fig. 11** Frequency distribution of solutions with fully developed chaos noise for Nug30.

**Table 4** *Depth_1* for Nug30.

| $D_\infty$ | *Depth_1* | | |
|---|---|---|---|
| | Intermittency chaos | Burst | Fully developed chaos |
| 10000 | 3.89e+10 | 2.48e+10 | 1.96e+09 |

From these results, we can say that the intermittency chaos noise and the burst noise generated by the Gilbert model has much better performance than the fully developed chaos noise and that there is still some difference between the intermittency chaos noise and the burst noise.

**Table 5** *Depth_2* for Nug30.

| | *Depth_2* | | |
|---|---|---|---|
| $D_{th}$ | Intermittency chaos | Burst | Fully developed chaos |
| 8536 | 6.04e+09 | 4.32e+09 | 3.23e+08 |
| 8134 | 2.14e+09 | 1.71e+09 | 1.21e+08 |
| 7732 | 2.78e+08 | 3.47e+08 | 1.99e+07 |

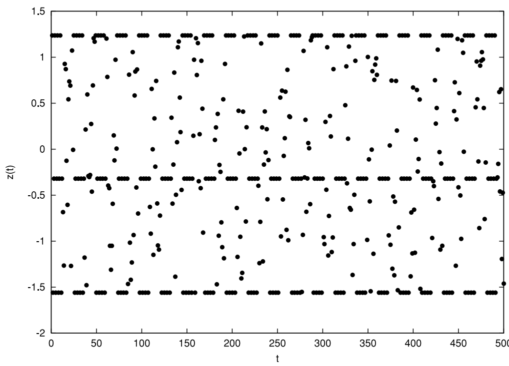


**Fig. 12** Regular burst noise.

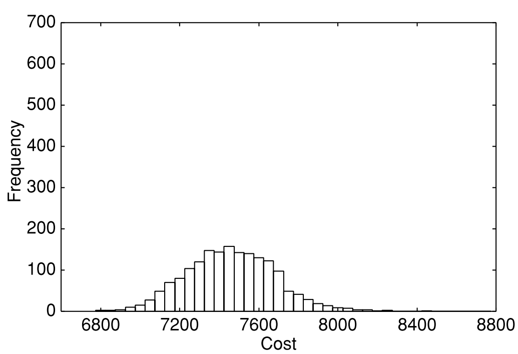


**Fig. 13** Frequency distribution of solutions with regular burst noise for Nug30.

### 4.5 Regular Burst Noise

At last, in order to support our conjecture on the good performance of the intermittency chaos, we consider a regular burst noise whose laminar parts and burst parts have fixed lengths and appear alternately. The length of the laminar parts and the burst parts are fixed as 4-period. Figure 12 shows an example of the regular burst noise and Fig. 13 shows the frequency distribution of solutions when the noise is applied to the network under the same conditions as 4.4.

As we can see from the frequency distribution, the regular switching of the laminar parts and the burst parts makes the performance worse remarkably. (The difference between Figs. 10 and 13 might looks similar to the difference between Figs. 9 and 10. However, the difference of the time series used for Figs. 10 and 13 is only the regularity of the switchings and it is considered to be much smaller than that of the time series used for Figs. 9 and 10.) Hence, we can conclude that not only the existence of the laminar parts and the burst parts but also their irregular switching is an important factor of the intermittency chaos.

## 5. Conclusions

In this paper, we have investigated the performance of the chaos and the burst noises injected to the Hopfield NN for QAP. For the correct evaluation of the noises, we proposed two methods to appreciate finding a lot of nearly optimal solutions. We confirmed by the computer simulations that the burst noise generated by the Gilbert model with a laminar part and a burst part achieved the good performance as the intermittency chaos noise near the three-periodic window. The result suggests that the irregular switching of the laminar parts and the burst parts is one reason of the good performance of the Hopfield NN with chaos noise. We also found that there is still some difference between the intermittency chaos noise and the burst noise. Investigating the reason of the difference in detail is our important future research.

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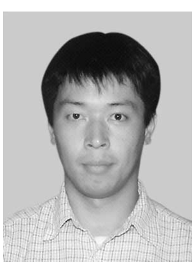

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