# Refined analysis and updated constraints on general non-standard $tbW$ couplings

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A B S T R A C T

We recently studied possible non-standard $tbW$ couplings based on the effective-Lagrangian which consists of four kinds of $SU(3) \times SU(2) \times U(1)$ invariant dimension-6 effective operators and gave an experimentally allowed region for each non-standard coupling. We here re-perform that analysis much more precisely based on the same experimental data but on a new computational procedure using the Graphics-Processing-Unit (GPU) calculation system. Comparing these two analyses with each other, the previous one is found to have given quite reliable results despite of its limited computation capability. We then apply this new procedure to the latest data and present updated results.

The top quark, the heaviest particle we have ever encountered up to now, is expected to play an important role as a window opened for a possible new physics beyond the standard model. The Large Hadron Collider (LHC) has been accumulating more and more data on this quark and will soon enable its precision studies. In our recent article [1], we performed an analysis of possible non-standard top–bottom–$W$ ($tbW$) couplings as model-independently as possible based on the effective-Lagrangian framework [2–5] by using available experimental data of top-decay processes at the LHC.[1]

The effective-Lagrangian we used there consists of $SU(3) \times SU(2) \times U(1)$ invariant operators whose mass-dimension is six, and there are four kinds of operators that could contribute to the $tbW$ couplings. There we have given allowed regions for those non-standard couplings. The precision level of the results was, however, not high enough due to its computational limitation. In this note, we aim to re-analyze the same experimental data but on a new computational procedure using the Graphics-Processing-Unit (GPU) calculation system. We will thereby be able to check how reliable the last analysis was. We then apply this procedure to the latest data and present more precise constraints on those couplings.

In our framework [1], assuming that there exists some new physics characterized by an energy scale $\Lambda$ (e.g., the mass of a typical new particle) and all the non-standard particles are not lighter than the LHC energy, the standard-model Lagrangian of $tbW$ interactions describing phenomena around the electroweak scale is extended as

$$\mathcal{L}_{tbW} = -\frac{1}{\sqrt{2}} g \Big[ \bar{\psi}_b(x) \gamma^\mu (f_1^L P_L + f_1^R P_R) \psi_t(x) W_\mu^-(x) + \bar{\psi}_b(x) \frac{\sigma^{\mu\nu}}{M_W} (f_2^L P_L + f_2^R P_R) \psi_t(x) \partial_\mu W_\nu^-(x) \Big],$$
(1)

where $g$ is the $SU(2)$ coupling constant, $P_{L/R} \equiv (1 \mp \gamma_5)/2$, and $f_{1,2}^{L,R}$ stand for the corresponding coupling parameters. Among those parameters, we divide $f_1^L$ into the SM term and the rest (i.e., the non-SM term) as

$$f_1^L \equiv f_1^{SM} + \delta f_1^L,$$
(2)

where we assume $f_1^{SM}(= V_{tb}) = 1$, and treat $\delta f_1^L$, $f_1^R$, and $f_2^{L/R}$ as non-standard complex couplings which are all independent of each other.

In order to give constraints on them, we use the following experimental information as our input data:

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[1] We have given a detailed list of preceding works by other authors in [1]. We would like to add [6] to the list, which has appeared after our work.

**Table 1**
Allowed maximum and minimum values of the non-standard-top-decay couplings in the case that all the couplings are dealt with as free parameters. Those in the parentheses show the previous results.

| | $\delta f_1^L$ | | $f_1^R$ | | $f_2^L$ | | $f_2^R$ | |
|---|---|---|---|---|---|---|---|---|
| | $\mathrm{Re}(\delta f_1^L)$ | $\mathrm{Im}(\delta f_1^L)$ | $\mathrm{Re}(f_1^R)$ | $\mathrm{Im}(f_1^R)$ | $\mathrm{Re}(f_2^L)$ | $\mathrm{Im}(f_2^L)$ | $\mathrm{Re}(f_2^R)$ | $\mathrm{Im}(f_2^R)$ |
| Min. | −2.58 | −1.58 | −1.36 | −1.36 | −0.68 | −0.68 | −1.20 | −1.20 |
| | (−2.55) | (−1.55) | (−1.30) | (−1.30) | (−0.65) | (−0.65) | (−1.20) | (−1.20) |
| Max. | 0.58 | 1.58 | 1.36 | 1.36 | 0.68 | 0.68 | 1.20 | 1.20 |
| | (0.55) | (1.55) | (1.30) | (1.30) | (0.65) | (0.65) | (1.20) | (1.20) |

**Table 2**
Allowed maximum and minimum values of non-standard-top-decay couplings in the case that all the couplings are dealt with as free parameters except for $\mathrm{Re}(\delta f_1^L)$ being set to be zero. Those in the parentheses show the previous results.

| | $\delta f_1^L$ | $f_1^R$ | | $f_2^L$ | | $f_2^R$ | |
|---|---|---|---|---|---|---|---|
| | $\mathrm{Im}(\delta f_1^L)$ | $\mathrm{Re}(f_1^R)$ | $\mathrm{Im}(f_1^R)$ | $\mathrm{Re}(f_2^L)$ | $\mathrm{Im}(f_2^L)$ | $\mathrm{Re}(f_2^R)$ | $\mathrm{Im}(f_2^R)$ |
| Min. | −1.23 | −1.14 | −1.12 | −0.55 | −0.57 | −0.96 | −1.00 |
| | (−1.20) | (−1.10) | (−1.10) | (−0.50) | (−0.55) | (−0.95) | (−1.00) |
| Max. | 1.23 | 1.10 | 1.12 | 0.59 | 0.57 | 0.00 | 1.00 |
| | (1.20) | (1.05) | (1.10) | (0.55) | (0.55) | (0.00) | (1.00) |

**Table 3**
Allowed maximum and minimum values of non-standard-top-decay couplings in the case that all the couplings are dealt with as free parameters except for $\mathrm{Re}(\delta f_1^L)$ and $\mathrm{Im}(\delta f_1^L)$ both being set to be zero. Those in the parentheses show the previous results.

| | $f_1^R$ | | $f_2^L$ | | $f_2^R$ | |
|---|---|---|---|---|---|---|
| | $\mathrm{Re}(f_1^R)$ | $\mathrm{Im}(f_1^R)$ | $\mathrm{Re}(f_2^L)$ | $\mathrm{Im}(f_2^L)$ | $\mathrm{Re}(f_2^R)$ | $\mathrm{Im}(f_2^R)$ |
| Min. | −1.14 | −1.12 | −0.55 | −0.57 | −0.96 | −0.49 |
| | (−1.10) | (−1.10) | (−0.50) | (−0.55) | (−0.95) | (−0.45) |
| Max. | 1.10 | 1.12 | 0.59 | 0.57 | 0.00 | 0.49 |
| | (1.05) | (1.10) | (0.55) | (0.55) | (0.00) | (0.45) |

- The total decay width of the top quark [7]

$$\Gamma^t = 1.36 \pm 0.02(\text{stat.})^{+0.14}_{-0.11}(\text{syst.})\ \text{GeV}.^2 \qquad (3)$$

- The partial decay widths derived from experimental data of $W$-boson helicity fractions [8] with the above $\Gamma^t$

$$\Gamma_L^{t*} = 0.405 \pm 0.072\ \text{GeV},$$
$$\Gamma_0^{t*} = 0.979 \pm 0.125\ \text{GeV}, \qquad (4)$$
$$\Gamma_R^{t*} = -0.024 \pm 0.030\ \text{GeV}.$$

Varying all the parameters at the same time, we look for the area in the parameter space in which we find solutions to satisfy the above input and outside of which any parameter values there do not reproduce the data. We then represent the resultant allowed region for each parameter by giving its maximum and minimum values. Throughout the computations, we do not neglect any contributions, i.e., we keep not only the SM term plus those linear in the non-standard couplings but also those quadratic in them.

In the previous work [1], the analysis was carried out by varying each parameter in steps of 0.05 using a workstation [ 67.2GFLOPS ]. Here we re-analyze the same data in order to see if we could give more precise constraints on each parameter using a GPU calculator [ 4.29TFLOPS ]. We take as $m_t = 172.5$ GeV, $m_b = 4.8$ GeV and $M_W = 80.4$ GeV for the masses of the involved particles as in [1].

The results corresponding to the previous ones are shown in Tables 1, 2 and 3: the allowed regions between the maximum and minimum in those tables have been obtained respectively from the eight-parameter analysis (i.e. all the parameters are treated as free ones) in steps of 0.02, the seven-parameter one (i.e. $\mathrm{Re}(\delta f_1^L) = 0$, the others are treated as free parameters) and the six-parameter one (i.e. $\mathrm{Re}(\delta f_1^L) = \mathrm{Im}(\delta f_1^L) = 0$, the others are treated as free parameters) both in 0.01 steps.³

From a general point of view, the maximum/minimum of the allowed region is expected to increase/decrease by up to 0.05 (0.04) if we change the step size from 0.05 to 0.01 (0.02). The actual changes of the boundaries are however smaller than this naive expectation except for that of $f_1^R$ in Table 1. The fact that most of them have not changed so much means that our previous analysis has already given quite reliable results despite of its rather large step size. In addition, the exceptional behavior of $f_1^R$ tells us that several parameters could interact with each other in analyses like the present one and consequently some parameters get larger allowed regions than we imagine. Let us note that this would never happen in a "multiple-parameter analysis" in which only one parameter is varied at once.

This way we have confirmed that our previous analysis is well reliable, but it does not mean that we have obtained nothing new in the present analysis. In order to show how the precision has been raised here, we give in Table 4 the increase rate of each allowed region in percentage. We see that this re-analysis has been worth performing especially for $f_1^R$ and $f_2^L$. It is also remarkable

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² In fact, it is not easy to handle an asymmetric error like this in the error propagation. We therefore use $\Gamma^t = 1.36 \pm 0.02(\text{stat.}) \pm 0.14(\text{syst.})$ GeV, the one symmetrized by adopting the larger (i.e., +0.14) in this systematic error.

³ It would take more than 12 years to get a meaningful result in an eight-parameter analysis in steps of 0.01, even if the GPU calculator were used. Therefore, we have adopted 0.02 steps for the eight-parameter analysis.

**Table 4**

The increase rates of the allowed regions compared with the previous results.

|  | $\delta f_1^L$ | | $f_1^R$ | | $f_2^L$ | | $f_2^R$ | |
|---|---|---|---|---|---|---|---|---|
|  | $\mathrm{Re}(\delta f_1^L)$ | $\mathrm{Im}(\delta f_1^L)$ | $\mathrm{Re}(f_1^R)$ | $\mathrm{Im}(f_1^R)$ | $\mathrm{Re}(f_2^L)$ | $\mathrm{Im}(f_2^L)$ | $\mathrm{Re}(f_2^R)$ | $\mathrm{Im}(f_2^R)$ |
| 8 param. | 1.9% | 1.9% | 4.6% | 4.6% | 4.6% | 4.6% | 0.0% | 0.0% |
| 7 param. | – | 2.5% | 4.2% | 1.8% | 8.6% | 3.6% | 1.1% | 0.0% |
| 6 param. | – | – | 4.2% | 1.8% | 8.6% | 3.6% | 1.1% | 8.9% |

**Table 5**

Updated constraints on the non-standard-top-decay couplings in the case that all the couplings are dealt with as free parameters.

|  | $\delta f_1^L$ | | $f_1^R$ | | $f_2^L$ | | $f_2^R$ | |
|---|---|---|---|---|---|---|---|---|
|  | $\mathrm{Re}(\delta f_1^L)$ | $\mathrm{Im}(\delta f_1^L)$ | $\mathrm{Re}(f_1^R)$ | $\mathrm{Im}(f_1^R)$ | $\mathrm{Re}(f_2^L)$ | $\mathrm{Im}(f_2^L)$ | $\mathrm{Re}(f_2^R)$ | $\mathrm{Im}(f_2^R)$ |
| Min. | −2.56 | −1.56 | −1.32 | −1.32 | −0.70 | −0.70 | −1.20 | −1.20 |
| Max. | 0.56 | 1.56 | 1.32 | 1.32 | 0.70 | 0.70 | 1.20 | 1.20 |

**Table 6**

Updated constraints on the non-standard-top-decay couplings in the case that all the couplings are dealt with as free parameters except for $\mathrm{Re}(\delta f_1^L)$ being set to be zero.

|  | $\delta f_1^L$ | $f_1^R$ | | $f_2^L$ | | $f_2^R$ | |
|---|---|---|---|---|---|---|---|
|  | $\mathrm{Im}(\delta f_1^L)$ | $\mathrm{Re}(f_1^R)$ | $\mathrm{Im}(f_1^R)$ | $\mathrm{Re}(f_2^L)$ | $\mathrm{Im}(f_2^L)$ | $\mathrm{Re}(f_2^R)$ | $\mathrm{Im}(f_2^R)$ |
| Min. | −1.20 | −1.12 | −1.10 | −0.57 | −0.59 | −0.96 | −1.00 |
| Max. | 1.20 | 1.07 | 1.10 | 0.61 | 0.59 | 0.00 | 1.00 |

**Table 7**

Updated constraints on the non-standard-top-decay couplings in the case that all the couplings are dealt with as free parameters except for $\mathrm{Re}(\delta f_1^L)$ and $\mathrm{Im}(\delta f_1^L)$ both being set to be zero.

|  | $f_1^R$ | | $f_2^L$ | | $f_2^R$ | |
|---|---|---|---|---|---|---|
|  | $\mathrm{Re}(f_1^R)$ | $\mathrm{Im}(f_1^R)$ | $\mathrm{Re}(f_2^L)$ | $\mathrm{Im}(f_2^L)$ | $\mathrm{Re}(f_2^R)$ | $\mathrm{Im}(f_2^R)$ |
| Min. | −1.12 | −1.10 | −0.57 | −0.59 | −0.96 | −0.49 |
| Max. | 1.07 | 1.10 | 0.61 | 0.59 | 0.00 | 0.49 |

that the increase rates for $\mathrm{Re}(f_2^R)$ are quite small in contrast to the other parameters. The reason will be that the change of $\mathrm{Re}(f_2^R)$ is not cancelled out by contributions from the other parameters, because that from $\mathrm{Re}(f_2^R)$ on the $tbW$ couplings is the largest one (except for the one from the standard model) since only this term can interfere with the standard-model term when the $b$-quark mass is neglected, i.e., all the other interference terms are proportional to $m_b$.

Now we know that our strategy and procedure are trustable and therefore we are ready to refine the analysis based on the following latest data on the partial decay widths:

- The partial decay widths derived from experimental data of $W$-boson helicity fractions [9] with $\Gamma^t$ in eq. (3)

$$\Gamma_L^{t*} = 0.439 \pm 0.051 \text{ GeV},$$
$$\Gamma_0^{t*} = 0.926 \pm 0.103 \text{ GeV},$$
$$\Gamma_R^{t*} = -0.005 \pm 0.020 \text{ GeV}.$$

(5)

We present the results of the eight-, seven- and six-parameter analyses in Tables 5, 6, and 7 respectively, which should be compared with those in Tables 1, 2 and 3. We see that there are meaningful improvements of 0.03–0.04 in a couple of results for $\delta f_1^L$ and $f_1^R$. Although some other boundaries have also changed, their sizes are of 0.02, which might come from the difference between the central values of (4) and (5) and therefore it is not easy yet to draw definite conclusion on them. In any case, these results are all consistent with the standard-model predictions, but it

is also noteworthy that there still exists enough space for a new physics beyond the standard model.

In conclusion, we have performed a re-analysis of the same experimental data as the previous one but on a new computational procedure, and presented more precise allowed regions of the non-standard $tbW$ couplings treated as complex numbers. There we find that the allowed regions have become slightly larger than those of the previous analysis. However, the overall tendencies of these two analyses seem to be consistent with each other. We therefore would like to stress that we have succeeded to raise the precision level through the re-analysis here and also that the previous analysis was better than we naively imagine from its limited-precision calculations. Having confirmed this way that our strategy and procedure are trustable, we then have made a new analysis based on the latest experimental data and given updated constraints on those couplings. There the constraints on $\delta f_1^L$ and $f_1^R$ have been further improved, and some other boundaries have also shown certain small changes.

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