

## PAPER

# Optical QPSK Signal Quality Degradation due to Phase Error of Pump Light in Optical Parametric Phase-Sensitive Amplifier Repeaters

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**SUMMARY** The influence of pump phase error on phase-sensitive optical amplifier (PSA) repeaters and the waveform degradation due to chromatic dispersion and fiber nonlinearities in the optical multi-relay transmission of quadrature phase-shift keying phase-conjugated twin waves are considered theoretically. First, the influence of noise from the pump phase error, optical local oscillator, receiver, and the amplified spontaneous-emission (ASE) in PSA repeaters is investigated with the assumption that transmission fibers are linear lossy channels. The bit-error rate (BER) is estimated as a function of the signal-to-noise ratio, and the relationship between the number of transmission relays and the fiber launch power is clarified. Waveform degradation due to chromatic dispersion and the optical fiber nonlinearities in transmission fibers are investigated with the noiseless condition, and the maximum repeatable number as a function of the fiber launch power is calculated. Finally, we show the relationship among the maximum repeatable number, standard deviation of pump phase error in PSA repeaters, and the fiber launch power to clarify the optimum transmission condition with consideration of the noise and the waveform degradation.

**key words:** phase-sensitive optical amplifier repeaters, pump phase error, phase-conjugated twin waves

## 1. Introduction

High-speed and large-capacity optical fiber communication systems must be implemented to satisfy the ever-increasing traffic demand and  $M$ -ary signaling, such as the quadrature phase-shift keying (QPSK) and 16-quadrature amplitude modulation (16-QAM), has been used for practical optical fiber transmission systems. However, as  $M$  increases, not only the number of bits in a transmitted symbol but also the required signal-to-noise ratio (SNR) increases [1]. The SNR is decreased by the amplified spontaneous-emission light (ASE) noise from erbium-doped optical fiber amplifier (EDFA) repeaters in optical multi-relay systems; as a result, the transmittable distance of  $M$ -ary signals is restricted. Additionally, high-order  $M$ -ary signals have a relatively high peak-to-average ratio (PAPR) and are severely distorted by optical fiber nonlinearities. Early studies including the optical phase conjugation [2] and digital back-propagation [3] have struggled with optical fiber nonlinearities and have not dealt with the SNR degradation due to ASE noise. On the

other hand, phase-conjugated twin-waves (PCTWs) transmission with frequency non-degenerate optical parametric phase-sensitive amplifier (PSA) repeaters [4] is a promising candidate to address the SNR degradation and optical fiber nonlinearities because the PSA repeaters achieve suppression of the nonlinear phase noise as well as low noise amplification.

A PSA is a particular type of optical parametric amplifier. PCTWs and a pump light are input to an optical parametric medium, such as a highly nonlinear fiber, and a periodically poled lithium niobate waveguide. For the PSA, the pump light needs to phase-lock to the average phase of the PCTWs. There are two approaches of pump phase-locking techniques. One is the optical injection-locking (OIL) technique [5], and the other is the optical phase-locked-loop (OPLL) technique [6], [7]. In the former, a pilot tone transmission is used and causes wavelength channel crosstalk in transmission fibers. PSA performance is also degraded because the transmitted pilot tone, which is affected by the optical fiber nonlinearities, is used as a pump light after the injection locking. Moreover, the OIL has a relatively wide bandwidth of 9.2 GHz [8], and the generated pump includes wide-band noise. This leads to pump light degradation, i.e., PSA performance degradation, due to the phase error of the pump light. In contrast, the OPLL does not require the pilot tone transmission; therefore, the transmission performance of 20-Gbit/s QPSK-PCTWs in optical multi-relay systems with PSA repeaters employing the OPLL is better than that in the OIL case [9]. Another advantage of OPLL over OIL is its incredibly low phase error, which is because the OPLL bandwidth is limited by a loop filter circuit with a bandwidth of at most several tens of MHz. However, a pump light generated by the OPLL still has a fractional phase error due to shot noise from the photodetectors and phase fluctuations in the optical fields resulting from spontaneous emission of laser sources [10] in an OPLL circuit. This would affect the PSA performance and transmission characteristics. The previous study [9] has not taken into account the pump phase error of PSA repeaters in optical multi-relay transmission of QPSK-PCTWs, and the influence of the pump phase error has not been clarified so far.

In this paper, we numerically investigate the influence of pump light phase error in PSA repeaters on optical multi-relay transmission of QPSK-PCTWs and clarify the required pump phase error that achieves a desired

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transmission distance. First, the relationship among pump phase errors in PSA repeaters, transmission distance, and fiber launch power is calculated with the assumption that a transmission fiber is a linear lossy channel. Next, an optical multi-relay transmission of 20-Gbit/s QPSK-PCTWs is simulated in view of the chromatic dispersion (CD) and optical fiber nonlinearities (FNL) in transmission fibers, and the transmission distance as a function of fiber launch power is calculated. Eventually, the maximum reach of 20-Gbit/s QPSK-PCTWs on optical multi-relay systems with PSA repeaters is found using the calculated results.

## 2. Investigation of Signal Degradation Due to Pump Light Phase Error and ASE in Linear Lossy Channel

In this section, the signal quality degradation of QPSK-PCTWs due to the pump light phase error and ASE noise from PSA repeaters is numerically evaluated. Waveform degradations caused by CD and FNL in transmission fibers are not taken into account in the calculations to focus on degradation from noise; therefore, the transmission fibers are treated as linear lossy channels in this section. In Sect. 2.1, signal degradation due to phase errors of pump light in PSA repeaters and an optical local oscillator (OLO) and receiver noise are considered. Then, in Sect. 2.2, ASE noise occurring in PSA repeaters is also taken into account as a deterioration factor.

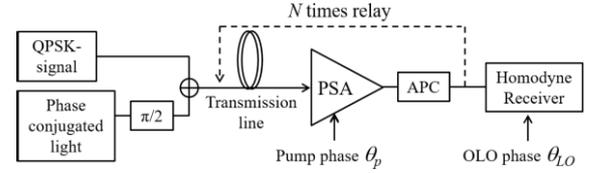
### 2.1 Signal Quality Degradation Due to Pump Light Phase Error

Figure 1 shows the simulation model of QPSK-PCTWs transmission in optical multi-relay systems with PSA repeaters. This calculation treated baseband symbols, i.e., 1 sample per symbol. First, the QPSK signal and its phase-conjugated signal were generated. Here, the phase-conjugated signal was phase-shifted by  $\pi/2$  to obtain the maximum gain in the PSA repeaters. Both signals, namely QPSK-PCTWs, were transmitted through the transmission fibers that were linear lossy channels and attenuated with 24-dB loss because this simulation assumed an 80-km fiber transmission per relay with 0.3-dB/km optical loss. After that, the transmitted QPSK-PCTWs were amplified by the PSA repeater simulated by the coupled-mode equations as follows.

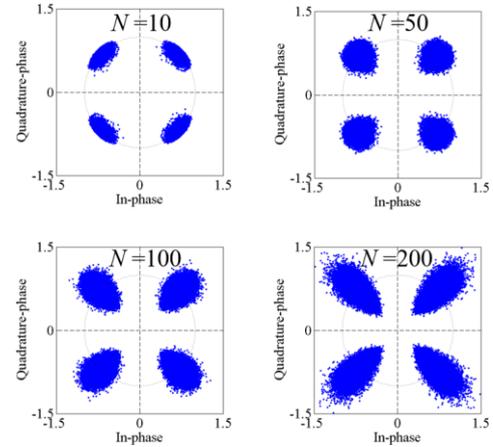
$$\phi_s(L) = \phi_s(0) \cosh \frac{gL}{2} + j\phi_c^*(0)e^{j2\theta_p} \sinh \frac{gL}{2} \quad (1)$$

$$\phi_c^*(L) = \phi_c^*(0) \cosh \frac{gL}{2} + j\phi_s(0)e^{-j2\theta_p} \sinh \frac{gL}{2} \quad (2)$$

Here,  $\phi_s$  and  $\phi_c$  are the complex amplitudes of the QPSK signals and the phase-conjugated signals.  $gL$  determines the PSA gain and was set to  $2.4 \ln(10)$  to obtain a 24-dB gain.  $\theta_p$  is the pump light phase with phase error standard deviation  $\sigma_{\theta_p}$ , which has a Gaussian distribution with 0 average. These formulas represent the input-output relation of



**Fig. 1** Simulation model of QPSK-PCTWs transmission in optical multi-relay systems with PSA repeaters. It is assumed that the pump phase  $\theta_p$  contains phase error standard deviation  $\sigma_{\theta_p}$ . APC is active power control.

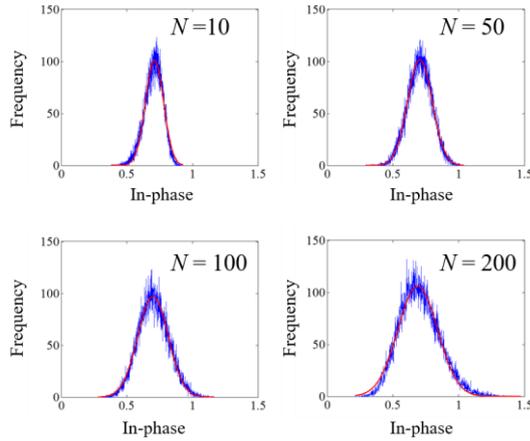


**Fig. 2** Constellation diagrams of the received signals are shown when the number of relays  $N$  is 10, 50, 100, and 200. Standard deviation of pump phase error  $\sigma_{\theta_p}$  was set to 90 mrad.

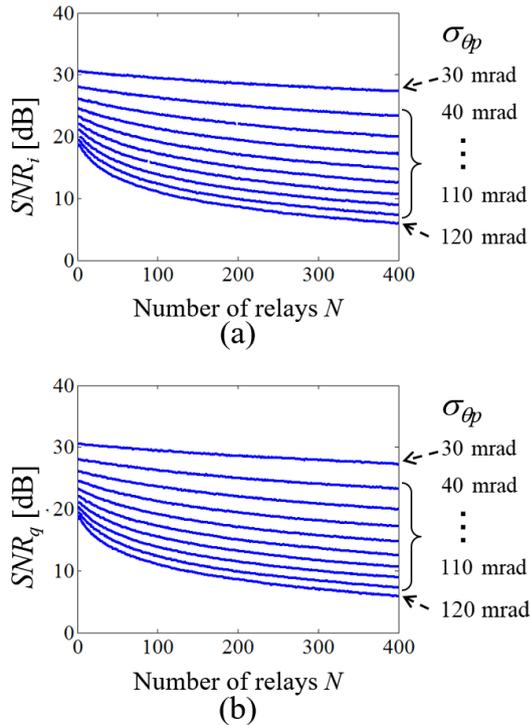
the PSA repeater and calculate the QPSK-PCTWs at the output of the PSA repeater. After the PSA repeater, the fraction optical power difference between the PSA output power and the fiber launch power was compensated by an active power controller (APC). Then, the QPSK-PCTWs were launched into the transmission fiber, amplified, and power-adjusted in the same manner, and these calculations were repeated  $N$  times. At the output of each relay, the QPSK signals were homodyne-detected and evaluated. Here, an optical local oscillator with  $\theta_{LO}$  included the phase error with standard deviation  $\sigma_{LO} (= \sigma_{\theta_p})$ .

Constellation diagrams of the received QPSK signals are shown in Fig. 2 when standard deviation of the pump phase error  $\sigma_{\theta_p}$  was set to 90 mrad, and the cases of  $N = 10, 50, 100,$  and  $200$  are depicted. As can be seen in Fig. 2, phase noise exists regardless of the number of relays  $N$ . However, the amplitude of the noise increases as the number of relays  $N$  increases. In principle, the parametric gain of PSAs decreases as the pump phase separates from the average phase of PCTWs. When the pump light includes phase error, the PSA output power momentarily decreases due to the instantaneous phase error, even though the average phase of pump light is equal to that of the PCTWs, i.e., the pump phase-locking state.

Histograms of in-phase components of received QPSK signals are shown in Fig. 3 and correspond to the constellation diagrams in Fig. 2. The blue lines show simulated data, and the red lines show fitting curves to the simulated



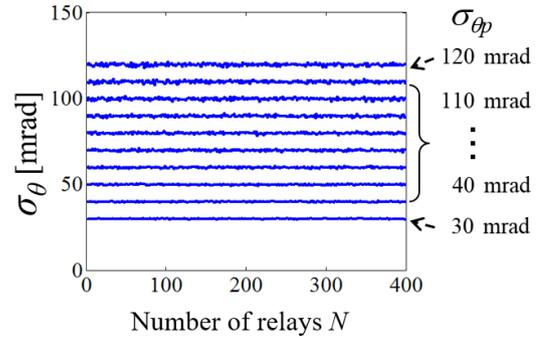
**Fig. 3** Histograms of received signals when the number of relays  $N$  is 10, 50, 100, and 200. Standard deviation of pump phase error  $\sigma_{\theta p}$  was set to 90 mrad. The blue lines indicate simulated data, and the red solid lines indicate fitting curves to the simulated data with the Gaussian function.



**Fig. 4** SNR characteristics as a function of the number of relays  $N$ . SNR was calculated for (a) in-phase and (b) quadrature-phase of received QPSK signals. The standard deviation of the pump phase error  $\sigma_{\theta p}$  ranges from 30 mrad to 120 mrad.

data with the Gaussian function. The simulated data is in good agreement with the Gaussian function, and the distribution of the histograms spreads as the number of relays  $N$  increases because of the pump phase fluctuation. For quantitative evaluation of the received QPSK signals, the SNR, defined as  $(\mu/\sigma)^2$ , was introduced and can be calculated by using the average  $\mu$  and the standard deviation  $\sigma$  from the fitted Gaussian curves.

Figure 4 shows the SNR characteristics as a function



**Fig. 5** Standard deviation of received QPSK signal phase as a function of the number of relays  $N$ . The standard deviation of the pump phase error  $\sigma_{\theta p}$  ranges from 30 mrad to 120 mrad.

of the number of relays  $N$ .  $SNR_i$  is the evaluated SNR for in-phase components, and  $SNR_q$  is the evaluated SNR for quadrature-phase components of the received QPSK signals. In both cases, SNR degrades as the number of relays  $N$  increases. This is because phase noise is converted to amplitude noise as can be seen in Fig. 2. The standard deviation of the received QPSK signals phase  $\sigma_{\theta}$  is also verified as a function of the number of relays  $N$ , as shown in Fig. 5. This indicates width of the signal phase histogram is not varied in accordance with the number of relays  $N$ . The standard deviation of the received QPSK signals  $\sigma_{\theta}$  does not depend on the number of relays  $N$  and is constant in spite of the standard deviation of the pump phase error  $\sigma_{\theta p}$ .

The receiver noise due to the shot noise, thermal noise, and phase noise resulting from the OLO in the homodyne receiver is taken into account. First, we define  $SNR_H$  determined by the arbitrary noise sources, except for the pump light phase error and the joint SNR for in-phase and quadrature-phase components, as follows.

$$SNR_I = \frac{1}{SNR_H^{-1} + SNR_i^{-1}} \quad (3)$$

$$SNR_Q = \frac{1}{SNR_H^{-1} + SNR_q^{-1}} \quad (4)$$

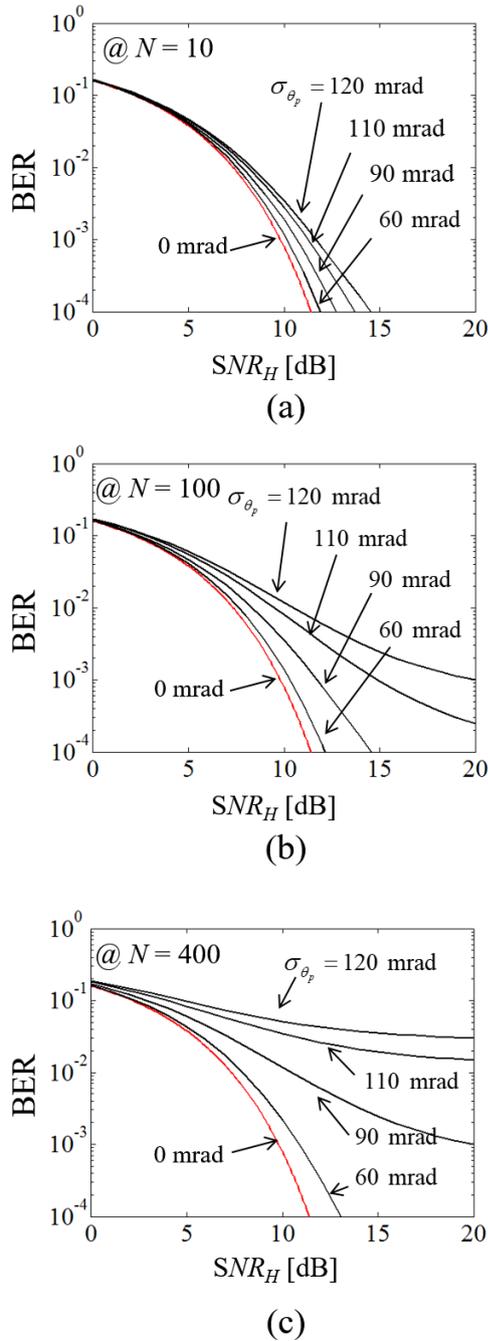
When the Gray coding is used, from Eqs. (3), (4), the theoretical BER as a function of OLO phase  $\theta_{LO}$ , which is the phase-noise difference between the signal and local sources, can be described as follows [11].

$$BER(\theta_{LO}) = \frac{1}{4} \operatorname{erfc} \left( \sqrt{SNR_I} \cos \left( \frac{\pi}{4} - \theta_{LO} \right) \right) + \frac{1}{4} \operatorname{erfc} \left( \sqrt{SNR_Q} \cos \left( \frac{\pi}{4} + \theta_{LO} \right) \right) \quad (5)$$

The expectation of the bit error rate (BER) for QPSK signals with influence from the OLO phase noise can be written as

$$E[BER] = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma_{\theta_{LO}}^2}} \exp \left( -\frac{\theta_{LO}^2}{2\sigma_{\theta_{LO}}^2} \right) \cdot BER(\theta_{LO}) d\theta_{LO}. \quad (6)$$

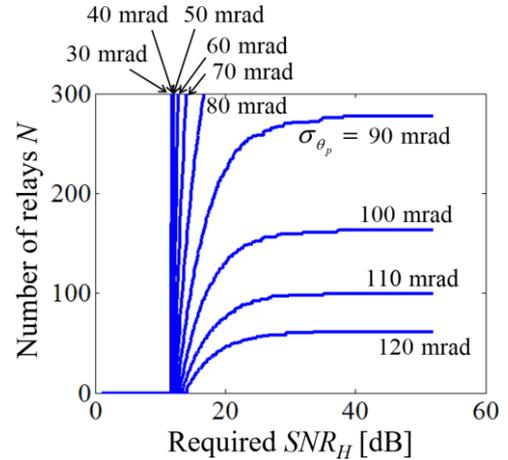
Here,  $E[\cdot]$  denotes the expectation operation.  $\sigma_{\theta_{LO}}$  is the standard deviation of the OLO phase error with the assumption



**Fig. 6** BER characteristics as a function of  $SNR_H$ . The number of relays  $N$  is (a) 10, (b) 100, and (c) 400. The standard deviation of the pump phase error  $\sigma_{\theta_p}$  ranges from 30 mrad to 120 mrad.

that it has the same performance as the pump light phase error, which has a Gaussian distribution with an average of 0 and standard deviation  $\sigma_{\theta_p}$ .

Theoretical curves of  $BER-SNR_H$  can be obtained from Eq. (6). The standard deviation of the pump phase error  $\sigma_{\theta_p}$  ranges from 0 mrad to 120 mrad, and Figs. 6(a)–(c) show the cases where the number of relays  $N$  are 10, 100, and 400. As can be seen in Fig. 6, the power penalty increases as the standard deviation of the pump phase error  $\sigma_{\theta_p}$  increases.



**Fig. 7** The number of relays  $N$  as a function of the required  $SNR_H$  dominated by the receiver noise and the OLO phase noise. The required  $SNR_H$  satisfies  $10^{-4}$ -BER, as shown in Fig. 6.

Additionally, the power penalty due to the pump phase error dominantly increases as the number of relays  $N$  increases. From the above results, we can summarize the relationship between the number of relays  $N$  and the required  $SNR_H$  as shown in Fig. 7. Here, the required  $SNR_H$  is  $SNR_H$  satisfying  $10^{-4}$ -BER. The required  $SNR_H$  increases along with the number of relays  $N$ , especially when the standard deviation of the pump phase error  $\sigma_{\theta_p}$  is equal to or greater than 90 mrad. From these results, the requirement of the pump phase error in PSA repeaters for long-haul transmission is theoretically clarified under the consideration of the pump phase error, receiver noise, and phase noise from the OLO in a linear lossy channel.

## 2.2 Signal Quality Degradation Due to ASE Noise at PSA Repeaters

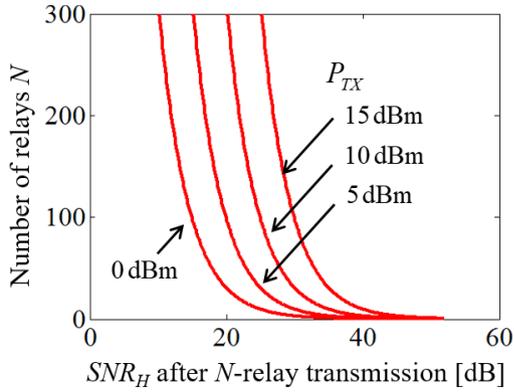
ASE noise generated in PSA repeaters severely affects the transmission distance in optical multi-relay systems owing to optical noise accumulation. In this subsection, the relationship between the signal quality and accumulated ASE is investigated. After  $N$  relays,  $SNR_H$  dominated by the ASE noise can be written as the following equation.

$$SNR_H = \frac{P_{TX}/h\nu}{2(G-1)N \cdot n_{sp} \cdot B} \quad (7)$$

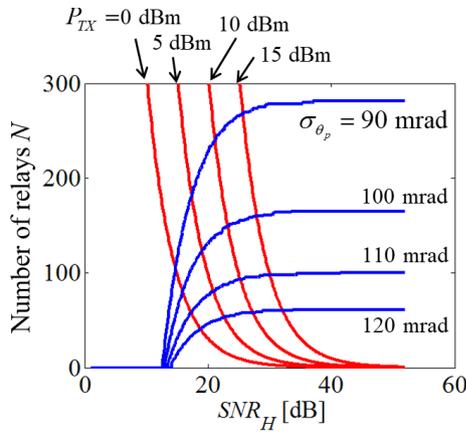
$P_{TX}$  is the fiber launch power,  $h$  is Planck's constant,  $\nu$  is the optical signal frequency,  $G$  is the amplifier gain,  $n_{sp}$  is the population inversion parameter, and  $B$  is the optical bandwidth. By deformation of Eq. (7), we obtain

$$N = \frac{P_{TX}/h\nu}{2(G-1) \cdot n_{sp} \cdot B \cdot SNR_H} \quad (8)$$

These formulas reveal that  $SNR_H$  decreases as  $N$  increases because of the ASE noise accumulation. Figure 8 illustrates the number of relays  $N$  as a function of  $SNR_H$  dominated by the ASE noise, and the parameters used for the calculation

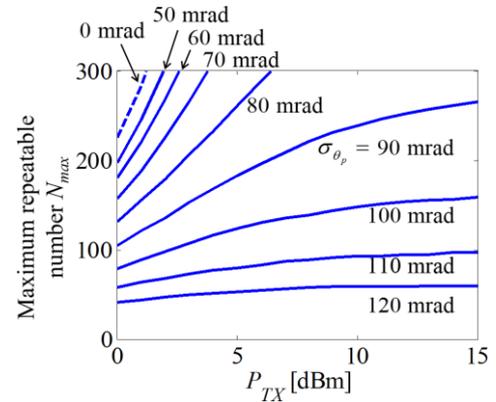


**Fig. 8** The number of relays  $N$  as a function of  $SNR_H$  dominated by ASE noise. The fiber launch power  $P_{TX}$  ranges from 0 to 15 dBm.



**Fig. 9** Superimposing Fig. 7 (blue curves) and Fig. 8 (red curves). When the fiber launch power  $P_{TX}$  and the standard deviation of the pump phase error  $\sigma_{\theta_p}$  are determined, the maximum repeatable number  $N_{max}$  can be found from the point of intersection in this graph with consideration of the accumulated ASE noise, receiver noise, OLO phase noise, and pump phase error.

are as follows:  $h = 6.626 \times 10^{-34} \text{ m}^2\text{kg/s}$ ,  $\nu = 193.5 \text{ THz}$ ,  $G = 24 \text{ dB}$ ,  $n_{sp} = 1$ , and  $B = 5 \text{ GHz}$ . The fiber launch power  $P_{TX}$  was also changed by 5 dB and ranged from 0 dBm to 15 dBm. Obviously,  $SNR_H$  improves as the fiber launch power increases, and degrades as the number of relays  $N$  increases resulting from the ASE noise accumulation. Then, the number of relays  $N$  as a function of the fiber launch power  $P_{TX}$  is calculated under the effect of the ASE noise accumulation, receiver noise, OLO phase noise, and pump phase error. Figure 9 is readily obtained by superimposing Fig. 7 and Fig. 8. When the fiber launch power  $P_{TX}$  and the standard deviation of the pump phase error  $\sigma_{\theta_p}$  are determined, the maximum repeatable number  $N_{max}$  can be found from the point of intersection, as shown in Fig. 9; as a result, the number of relays  $N$  as a function of the fiber launch power  $P_{TX}$  can be obtained as shown in Fig. 10. Note the maximum repeatable number  $N_{max}$  satisfying  $10^{-4}$ -BER. As can be seen from Fig. 10, a higher fiber launch power  $P_{TX}$  can increase the maximum repeatable number  $N_{max}$ . Furthermore, the smaller standard deviation of the pump light



**Fig. 10** The number of relays  $N$  as a function of the fiber launch power  $P_{TX}$ . The accumulated ASE noise, receiver noise, OLO phase noise, and pump light phase error in the PSA repeaters are taken into account. The standard deviation of the pump phase error  $\sigma_{\theta_p}$  ranges from 0 to 120 mrad.

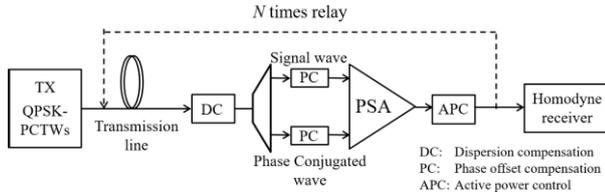
phase error  $\sigma_{\theta_p}$  can increase the maximum repeatable number  $N_{max}$ . From this result, it is clear that the pump phase error in PSA repeaters severely affects the maximum repeatable number  $N_{max}$  in a linear lossy channel as a linear transmission channel.

### 3. Signal Quality Degradation Due to Waveform Degradation from CD and FNL

In the previous section, signal quality degradation resulting from noise such as the ASE noise, receiver noise, OLO phase noise, and pump phase error in the linear transmission system was studied. In this section, waveform degradation of QPSK-PCTWs caused by CD and FNL is studied through numerical simulations. First, the numerical simulation model is shown and described. Next, the signal quality of received QPSK signals is evaluated, and the maximum repeatable number as a function of the fiber launch power  $P_{TX}$  is found.

#### 3.1 The Simulation Model of Waveform Degradation on QPSK-PCTWs Transmission in Multi-Relay System

Figure 11 shows the simulation model of QPSK-PCTWs transmission on optical multi-relay systems. First, in the transmitter, 20-Gbit/s electric QPSK signals were generated and frequency-shifted by +50 GHz. Here, a  $2^{10} - 1$  pseudorandom bit stream was used for data transmission, and the baseband QPSK signals were filtered by a cosine roll-off filter with a roll-off factor  $\alpha = 1$ . A continuous wave with 1.55- $\mu\text{m}$  center wavelength was then intensity-modulated by the electric QPSK signals with 50-GHz intermediate frequency, and the optical QPSK signal and its phase-conjugated signal, i.e., 20-Gbit/s QPSK-PCTWs, were generated with 100-GHz frequency spacing. The phase-conjugated wave was phase-shifted by  $\pi/2$  to maximize the amplifier gain in the PSA repeaters. The generated QPSK-PCTWs were launched into the transmission fiber with a length of 80 km. Light propagation in the fiber



**Fig. 11** Simulation model of 20-Gbit/s QPSK-PCTWs transmission on optical multi-relay systems. Transmission fibers are simulated by solving the nonlinear Schrödinger equation involving the effects of CD and FNL.

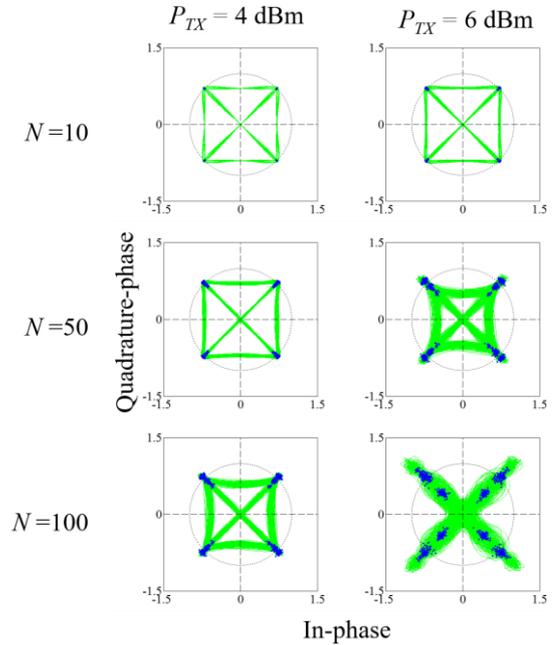
was simulated by solving the nonlinear Schrödinger equation expressed by

$$\frac{\partial A}{\partial z} = -\frac{1}{2}\alpha A - \frac{j}{2}\beta_2 \frac{\partial^2 A}{\partial T^2} + \frac{1}{6}\beta_3 \frac{\partial^3 A}{\partial T^3} + j\gamma|A|^2 A, \quad (9)$$

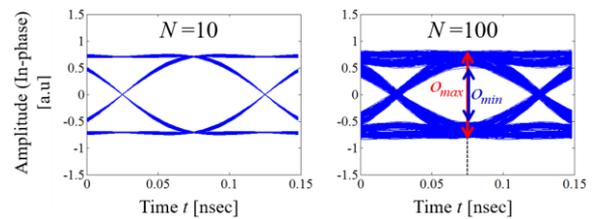
where  $A$  is the complex amplitude of the propagating light,  $z$  is the propagation length, and  $T$  is the relative time.  $\alpha$ ,  $\beta_2$ ,  $\beta_3$ , and  $\gamma$  represent the attenuation coefficient, second-order dispersion, third-order dispersion, and nonlinear coefficient, respectively. In this calculation, the following parameters were used for the transmission fiber:  $\alpha = 0.3$  dB/km,  $\gamma = 1.92$  /W/km, zero-dispersion wavelength  $\lambda_0 = 1.55$   $\mu$ m, and the dispersion slope  $D_s = 0.06$  ps/nm<sup>2</sup>/km. The transmitted QPSK-PCTWs were then input to the dispersion compensation fiber with ideal parameters, such as a 1-km fiber length, a 0-dB/km attenuation coefficient, and a 0-/W/km nonlinear coefficient. The zero-dispersion wavelength was also 1.55  $\mu$ m, and the dispersion slope was  $-0.06 \times 80$  ps/nm<sup>2</sup>/km. After the dispersion compensation fiber, the transmitted QPSK-PCTWs were divided into a signal wave and its phase-conjugated wave by a WDM demultiplexer with a 50-GHz rectangular bandpass characteristic. Next, each signal was phase-adjusted to compensate for the phase rotation due to self-phase modulation and input to the PSA repeaters. The PSA repeaters were simulated using Eqs. (1), (2), as in the previous section. Note that, because the waveform degradation resulting from the transmission fibers is focused in this section, the pump light phase error was neglected and  $\theta_p$  was set to zero. After  $N$  relays, the QPSK signals were input to the homodyne receiver with the assumption that the receiver did not have any noise, e.g., the receiver noise and the OLO phase noise. Finally, the received signal quality was confirmed.

### 3.2 Numerical Results of Signal Quality Degradation Caused by CD and FNL

Figure 12 shows the constellation diagrams of the received QPSK signals. Blue dots are received symbols. The left column is the case of  $P_{TX} = 4$  dBm, and the right column shows the case of  $P_{TX} = 6$  dBm. The upper row indicates the number of relays  $N = 10$ , the middle row indicates  $N = 50$ , and the lower row indicates  $N = 100$ . In each condition, phase fluctuation is suppressed by the PSA repeaters. However, the amplitude noise increases as the number of relays  $N$  increases. This tendency can be seen when the fiber

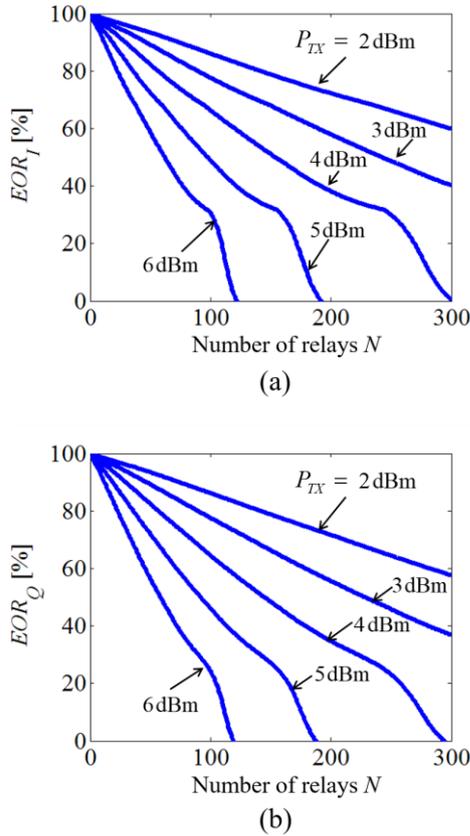


**Fig. 12** Constellation diagrams of received QPSK signals. Blue dots are received symbols, and green curves are trajectories of the symbols. The left column is the case of  $P_{TX} = 4$  dBm, and the right column shows the case of  $P_{TX} = 6$  dBm. The upper row indicates the number of relays  $N = 10$ , the middle row indicates  $N = 50$ , and the lower row indicates  $N = 100$ .

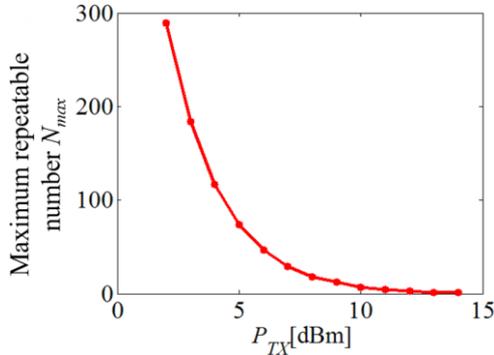


**Fig. 13** Eye diagram of received QPSK signals (In-phase). The fiber launch power  $P_{TX} = 4$  dBm. The cases of  $N = 10$  and  $N = 100$  are shown.

launch power  $P_{TX}$  increases, as shown in the right column of Fig. 12. Eye diagrams of the in-phase components are also confirmed and shown in Fig. 13. As in the constellation diagrams, the signal waveform is distorted with the increasing number of relays  $N$ . For quantitative evaluation of the received signal quality, the eye-opening ratio (EOR) defined as  $EOR = O_{min}/O_{max}$  was employed. Here,  $O_{min}$  and  $O_{max}$  are the amplitudes of the most inner and outer eye opening at the center of waveforms as shown in the right eye-pattern of Fig. 13. Figure 14 shows the EOR as a function of the number of relays  $N$  for in-phase and quadrature-phase components of the received signals. EOR degrades along with the increase in the number of relays  $N$  and the fiber launch power  $P_{TX}$  because of FNL. Here, the maximum repeatable number  $N_{max}$  is defined as the maximum number of relays  $N$  satisfying 60% EOR for both in-phase and quadrature-phase components in Fig. 14. Then, the maximum repeatable number  $N_{max}$ , as a function of the fiber launch power  $P_{TX}$ , is obtained as depicted in Fig. 15. The maximum repeatable number  $N_{max}$  drastically decreases as the fiber launch power

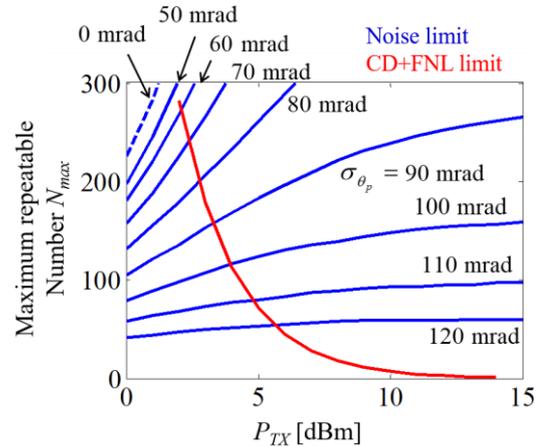


**Fig. 14** Eye opening ratio as a function of the number of relays  $N$  for (a) in-phase and (b) quadrature-phase of received signals. The fiber launch power  $P_{TX}$  ranges from 2 to 6 dBm.



**Fig. 15** Maximum repeatable number  $N_{max}$  as a function of the fiber launch power  $P_{TX}$ .  $N_{max}$  is defined as the maximum number of relays  $N$  satisfying 60% EOR for both in-phase and quadrature-phase components in Fig. 14.

$P_{TX}$  increases. This is because FNL mainly distorts the transmitted QPSK-PCTWs. From these results, the relationship between the maximum repeatable number  $N_{max}$  and the fiber launch power is numerically clarified from the standpoint of the waveform degradation due to CD and FNL.



**Fig. 16** Maximum repeatable number  $N_{max}$  as a function of the fiber launch power  $P_{TX}$ . This figure can be obtained by superimposing Fig. 10 (blue curves) and Fig. 15 (red curves). The intersection points of the noise limit with blue curves and the waveform degradation limit with red curves show the optimum fiber launch power for each standard deviation of the pump phase error  $\sigma_{\theta_p}$ .

#### 4. Optimum Transmission Condition Regarding the Pump Phase Error and the Fiber Launch Power in Optical Multi-Relay Systems with PSA Repeaters

We find the optimum transmission conditions of 20-Gbit/s QPSK-PCTWs in optical multi-relay systems with PSA repeaters in consideration of the noise and the waveform degradation due to CD and FNL, and clarify the relationship among the maximum repeatable number, pump phase error, and fiber launch power. Figure 16 is easily obtained by superimposing Fig. 10 in Sect. 2 and Fig. 15 in Sect. 3. The intersection points of the noise limit with blue curves and the waveform degradation limit with red curves show the optimum fiber launch power for each standard deviation of the pump phase error  $\sigma_{\theta_p}$ . For example, when the standard deviation of the pump phase error  $\sigma_{\theta_p}$  is 100 mrad in PSA repeaters, the maximum repeatable number  $N_{max}$  for 20-Gbit/s QPSK-PCTWS in optical multi-relay systems with PSA repeaters can be achieved over 100 times with 4-dBm fiber launch power. These calculation results are promising for the design of PSA repeaters employed in optical multi-relay systems for PCTWs transmission to meet system requirements.

#### 5. Conclusion

In this paper, the signal quality degradation of QPSK-PCTWs in optical multi-relay systems with PSA repeaters was investigated from the perspective of the noise. First, the noise due to the pump phase error, receiver noise, OLO phase noise, and ASE noise were taken into account under the assumption of a linear lossy channel. The estimated BER was found as a function of  $SNR_H$  and the number of relays  $N$  as a function of the fiber launch power. The waveform degradation caused by the chromatic dispersion and

the optical fiber nonlinearities was taken into account, and the maximum repeatable number  $N_{max}$  as a function of the fiber launch power was clarified through a numerical simulation of 20-Gbit/s QPSK-PCTWs transmission in optical multi-relay systems. Finally, we showed the waveform degradation and clarified the relationship among the maximum repeatable number, standard deviation of the pump phase error, and fiber launch power. It is expected that these results will contribute to the design of PCTWs multi-relay systems using PSA repeaters with pump phase-locking technique.

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